Wave-Based Damage Identification in Composite Strips and 2D Solids Using Inverse Multiresolution Wavelet Methods

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ABSTRACT

An inverse procedure for damage identification on 1D and 2D solids based on wave propagation using the multiresolution finite wavelet domain (MR-FWD) method is presented. The forenamed method utilizes Daubechies wavelet and scaling functions for the approximation of state variables and as such, it involves two types of solutions, the coarse and the fine solutions. In that way, the multiresolution nature of the method can be utilized for efficient damage estimation in experimental applications since the fine solutions of the method have manifested remarkable localization and isolation capabilities and high sensitivity to damage. In order to fully take advantage of the additional benefits of the MR-FWD method, full-field displacement measurements of the wave propagation are taken into consideration. Wavelet decomposition using Daubechies wavelets is now applied on the measurements, leading to approximation and detail components that are directly comparable to the coarse and fine solutions of the multiresolution simulation, respectively. Therefore, MR-FWD models can be created using the same Daubechies wavelets as the decomposition of the experimental data, so as to compare the simulation results with the measured ones. Numerical results reveal that comparing the detail component of the experiments with the fine solution of the simulations using appropriate metrics can lead to efficient damage identification. In such manner, an optimization process can be conducted in order to characterize the investigated damage scenarios. This procedure can lead to more sensitive and accurate damage estimation due to the advantages of the multiresolution analysis.

INTRODUCTION

The structural components get more sophisticated as time goes on and demands in industries like aerospace, maritime and infrastructure continue to rise. Structural Health Monitoring (SHM) is a process that involves gathering and interpreting data from a system of sensors that measure the structural response in order to objectively assess the condition of the structure. The main objective of a SHM system is to identify damage that could eventually lead to the failure of a particular component at an early stage. The findings of possible defects that are discovered through monitoring may subsequently

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be utilized to inform decisions for corrective actions [1]. Due to the low operational cost, the ability to scan broad areas and the capacity to identify small defects, wave-based SHM is a significant category of SHM procedures that is extensively explored [2]. Guided waves have been used for damage imaging utilizing many algorithms such as the delay-and-sum [3] and time reversal [4] techniques.

This paper focuses on the improved capabilities of the multiresolution finite wavelet domain method (MR-FWD) in the development of an inverse methodology employing model update to provide damage estimation using wave propagation in composite strips and 2D solids. The MR-FWD method utilizes both the scaling and wavelet functions of the Daubechies wavelets as basis functions [5], forming a hierarchical set of equations of motion. Concerning 1D models, the multiresolution (MR) approximation involves two solution types: the coarse and the fine solutions. When it comes to 2D models, the MR approach entails four solutions: the coarse solution and three fine solutions. The MR-FWD method has demonstrated outstanding computing advantages in transient dynamic simulations of rods, 2D solids [6], Timoshenko beams [7], and layerwise strips [8]. In order to benefit from the enhanced sensitivity of the fine solutions, the method's multiple resolution components are utilized in the model update process [9]. Wave response full-field measurements are considered available for demonstration purposes. Following the multiresolution decomposition [10], the measurements are decomposed to produce the approximation and detail components of the measurements, which are analogous to the coarse and fine solutions of the MR-FWD approach, respectively. This paper explores the potential of the proposed inverse methodology and its higher sensitivity in the parameter estimation.

THEORETICAL BACKGROUND

MR-FWD Method For 1D Models

The 1D MR reconstruction & decomposition approaches are shown in Figure 1.

The 1D generalized displacement approximation for R resolutions regarding the MR-FWD method is expressed as:

$$u(x,t) = \sum_{n=-(2L-2)}^{0} \hat{u}_{Cn}^{0}(t)\varphi(\xi-n) + \sum_{n=-(2L-2)}^{0} \{\sum_{S=0}^{R} \hat{u}_{Fn}^{S}(t)\psi(2^{s}\xi-n)\} \quad (1)$$

where $\hat{u}_{Cn}^{0}$ are the coarse wavelet coefficients at resolution 0, $\hat{u}_{Fn}^{S}$ are the fine wavelet coefficients at resolution S. Also, φ is the Daubechies scaling function (SF) and ψ is the Daubechies wavelet function (WF).

Single-Resolution or Resolution 0 (C^0). The C^0 solution is obtained employing only the DB SFs. The equation of motion is:

$$[M_{CC}]\ddot{\hat{u}}_{CC}(t) + [K_{CC}]\hat{u}_{CC}(t) = F_{C}(t) \quad (2)$$

where $\hat{u}_{CC}$ are the wavelet coefficients of the coarse approximation, $[K_{CC}]$ and $[M_{CC}]$ are the coarse resolution stiffness & mass matrices, and F_C is the coarse load vector.

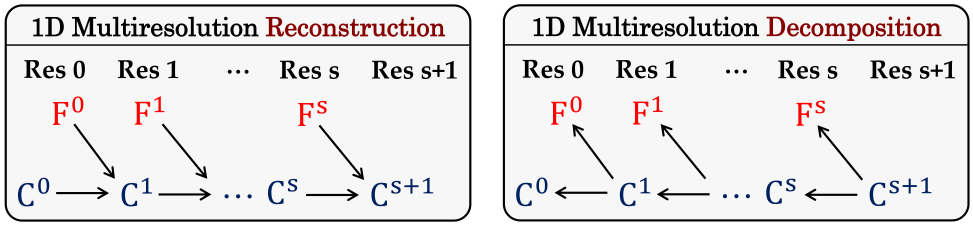

Figure 1. Schematic representation of 1D multiresolution reconstruction and decomposition process.

Resolution 1 (C^1). According to the MR process (Figure 1) the MR solution system is:

$$\begin{bmatrix} M_{CC} & 0 \\ 0 & M_{FF} \end{bmatrix} \begin{Bmatrix} \ddot{\hat{u}}_C(t) \\ \ddot{\hat{u}}_F(t) \end{Bmatrix} + \begin{bmatrix} K_{CC} & K_{CF} \\ K_{FC} & K_{FF} \end{bmatrix} \begin{Bmatrix} \hat{u}_C(t) \\ \hat{u}_F(t) \end{Bmatrix} = \begin{Bmatrix} F_C(t) \\ F_F(t) \end{Bmatrix} \tag{3}$$

where $\hat{u}_F$ are the fine wavelet coefficients, $\hat{u}_C$ are the coarse wavelet coefficients for resolution 1, $[K_{FF}]$ and $[M_{FF}]$ are the fine resolution stiffness & mass matrices and F_F is the fine resolution load vector. Also, $\hat{u}_C$ is not equal to $\hat{u}_{CC}$ because of the stiffness coupling terms, $[K_{CF}]$ and $[K_{FC}]$. However, $[M_{CF}]$, $[M_{FC}]$ are equal to zero and $[M_{CC}]$, $[M_{FF}]$ are diagonal due to the orthogonality of SFs/ WFs.

MR-FWD Method For 2D Models

The 2D MR reconstruction & decomposition approaches are visualized in Figure 2. The 2D generalized displacement approximation for R resolutions is given as:

$$\begin{aligned} u(x,y,t) = & \sum_{k,l=-(2L-2)}^{0} \hat{u}_{CC,kl}(t)\cdot\ \phi(\xi-k)\phi(\eta-l) + \sum_{k,l=-(2L-2)}^{0} \sum_{s=0}^{R} \hat{u}^s_{CF,kl}(t)\cdot\ \phi(2^s\xi-k)\psi(2^s\eta-l) \\ & + \sum_{k,l=-(2L-2)}^{0} \sum_{s=0}^{R} \hat{u}^s_{FC,kl}(t)\cdot\ \psi(2^s\xi-k)\phi(2^s\eta-l) + \sum_{k,l=-(2L-2)}^{0} \sum_{s=0}^{R} \hat{u}^s_{FF,kl}(t)\cdot\ \psi(2^s\xi-k)\psi(2^s\eta-l) \end{aligned} \tag{4}$$

where $\hat{u}_{CC}$ is the coarse component, while $\hat{u}_{CF}$, $\hat{u}_{FC}$ and $\hat{u}_{FF}$ are the fine components; *CF* is the vertical detail, *FC* is the horizontal detail and *FF* is the diagonal detail component. For resolution 1, the MR solution system is given as:

$$\begin{bmatrix} M_{CCCC} & 0 & 0 & 0 \\ 0 & M_{CCFF} & 0 & 0 \\ 0 & 0 & M_{FFCC} & 0 \\ 0 & 0 & 0 & M_{FFFF} \end{bmatrix} \begin{Bmatrix} \ddot{\hat{U}}_{CC} \\ \ddot{\hat{U}}_{CF} \\ \ddot{\hat{U}}_{FC} \\ \ddot{\hat{U}}_{FF} \end{Bmatrix} + \begin{bmatrix} K_{CCCC} & K_{CCCF} & K_{CFCC} & K_{CFCF} \\ K_{CCFC} & K_{CCFF} & K_{CFFC} & K_{CFFF} \\ K_{FCCC} & K_{FCCF} & K_{FFCC} & K_{FFCF} \\ K_{FCFC} & K_{FCFF} & K_{FFFC} & K_{FFFF} \end{bmatrix} \begin{Bmatrix} \hat{U}_{CC} \\ \hat{U}_{CF} \\ \hat{U}_{FC} \\ \hat{U}_{FF} \end{Bmatrix} = \begin{bmatrix} F_{CC} \\ F_{CF} \\ F_{FC} \\ F_{FF} \end{bmatrix} \tag{5}$$

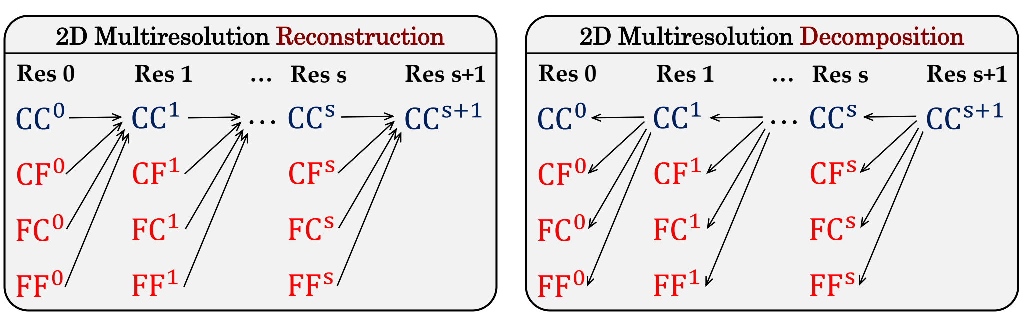

Figure 2. Schematic representation of 2D multiresolution reconstruction and decomposition process.

INVERSE DAMAGE ESTIMATION APPROACH

The suggested inverse damage estimating process utilizes the MR-FWD method's various resolution components to benefit from the great sensitivity of its fine solutions, whether in 1D or 2D scenarios. To accomplish this, it is also necessary to decompose the experimental data using the same SF/WF and resolution in order to obtain the approximation and detail components of the data, which are directly comparable to the coarse and fine solutions of the MR simulation, respectively.

Methodology For 1D Models

The proposed methodology for 1D scenarios requires the wave experiment full-field data. These data are decomposed into separate components for each time step using a specific Daubechies SF/WF and resolution level. The approximation and the detail component of the experimental data are thus acquired as two datasets. Then, utilizing the MR-FWD approach for the simulations, an optimization process can be carried out to employ the coarse and fine solutions, which are directly comparable to the approximation and detail components. The design variables can be estimated by using suitable objective functions, and so, the damage scenario can be evaluated. The inverse damage estimation procedure is schematically presented in Figure 3.

Methodology For 2D Models

The proposed methodology for 2D scenarios requires a snapshot of full-field data. The process is approximately the same as in the 1D cases and is illustrated in Figure 4.

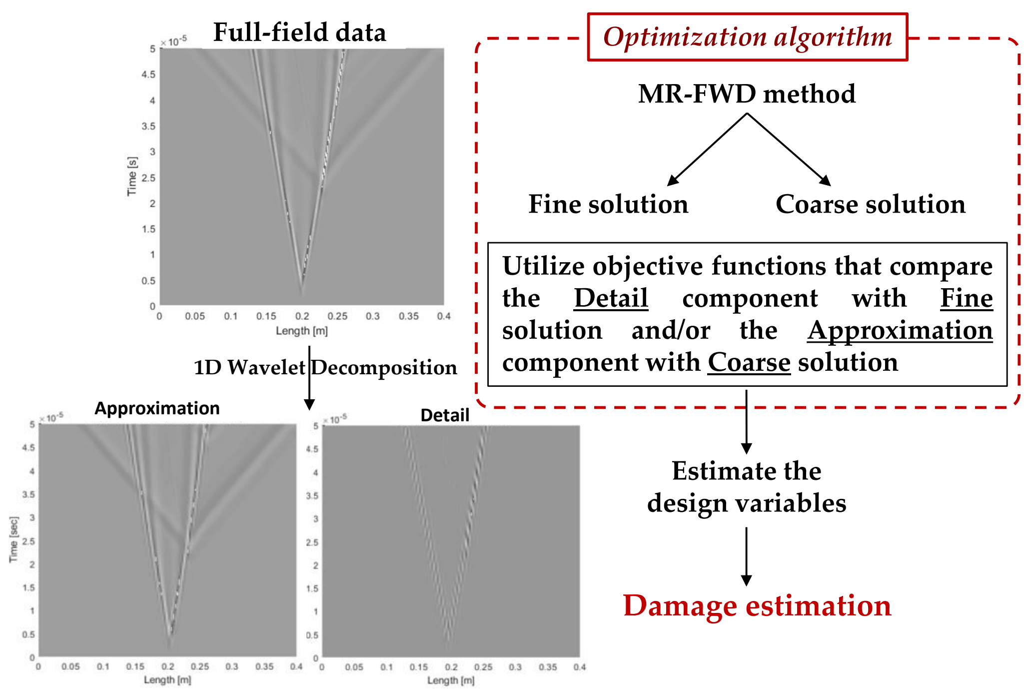

Figure 3. Damage estimation process using the MR-FWD method for 1D cases.

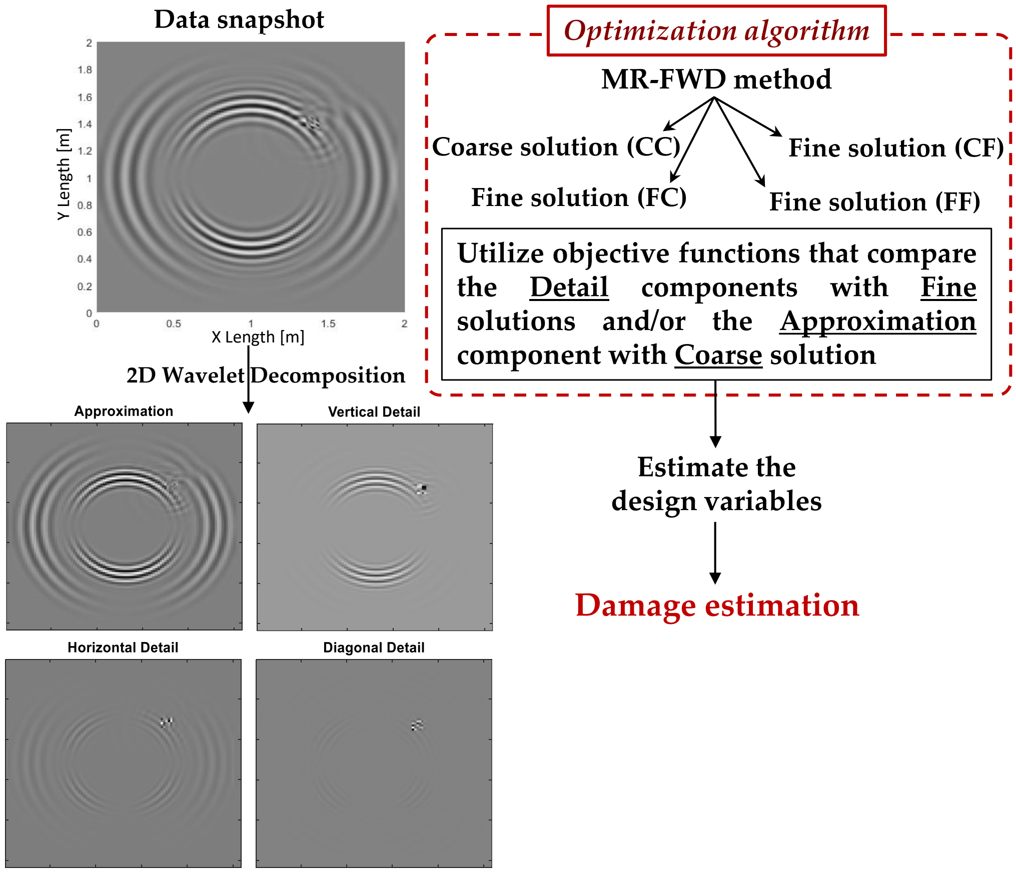

Figure 4. Damage estimation process using the MR-FWD method for 2D cases.

NUMERICAL RESULTS

1D Scenario

In this paper, the "pseudo-experimental" data are obtained from simulations in order to show the potential of the proposed methodology. The studied structure is a damaged composite strip, with geometric and model characteristics that are shown in Figure 5. The strip consists of unidirectional CFRP with material properties shown in TABLE I. Also, the analysis duration is 0.05 ms. A single-resolution FWD analysis (C^0) is utilized to produce the pseudo-experimental data using 460 DB6 elements.

TABLE I. MATERIAL PROPERTIES

	E_{11} (GPa)	$E_{22}=E_{33}$ (GPa)	$G_{12}=G_{23}=G_{13}$ (GPa)	$v_{12}=v_{13}$	v_{23}	ρ (kg/m^3)
CFRP	120	7.9	3.4	0.275	0.15	1578

The vertical displacement at the beam's top surface is the quantity that is considered as measured by a full-field scanning vibrometer. Those measured data are decomposed in each time step using wavelet decomposition in one resolution with the Daubechies DB6 wavelet and so, an approximation and a detail dataset have occurred. Therefore, the utilized objective functions employed in the model update process consist of metrics that compare the detail component of the pseudo-experimental data to the fine solution of the MR-FWD simulations. Two objective functions are used: the first compares the envelopes of the two forenamed datasets (detail component vs fine solution) and the second compares the wavenumber spectra of the two datasets at the last time step.

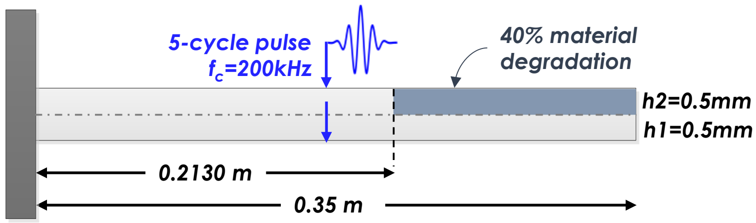

Figure 5. Geometric representation of the damaged strip, with the damage shown in dark gray color.

Determining the design variables is necessary before beginning the optimization process. The thickness ratio of the damaged span and the severity of the damage will essentially be the design factors in this case study. The coefficient that is multiplies all the elastic constants to produce the damaged material is the first design variable, X1. The proportion of the height, *h1*, to the overall height, (*h1*+*h2*), is the second design variable, or X2. TABLE II displays the lower and upper bounds for the design variables.

TABLE II. DESIGN VARIABLES AND THEIR LOWER AND UPPER BOUNDS

Design Variables	Lower Bound	Upper Bound
X1	0.1	1
X2	0.1	0.9

In this work, the Simulated Annealing (SA) algorithm [11] is used as a metaheuristic method to address the relevant optimization issue. Using MATLAB® R2019b on a laptop with an Intel® Core i7-9750H @ 2.60 GHz CPU and 16 GB RAM, several metaheuristic algorithms were tested before the SA method was chosen because of its much quicker results. Due to the stochastic and single-solution character of the SA algorithm, it should be noted that multiple runs have been carried out with various initial values of the design variables. The results of the inverse methodology are displayed in TABLE III. Two different approaches are evaluated. The first one, designated as A1, uses the detail component of the pseudo-experimental signals and the fine solution of the MR-FWD models, in order to demonstrate the higher sensitivity of the fine solutions. The second approach, termed as A2, is similar to the conventional model-based SHM methods since it directly compares the simulated results to the experimental ones. In TABLE III, the correct values and the predicted values of the SA algorithm are presented, as also their percentage differences (PD) for each design variables.

TABLE III. RESULTS OF THE MODEL UPDATE PROCEDURE

	X1	X2	PD w.r.t. X1	PD w.r.t. X2
Correct values	0.6	0.5	0%	0%
A1 approach	0.5804	0.4988	3.2667%	0.24%
A2 approach	0.5765	0.5246	3.9167%	4.92%

Both approaches clearly produce adequate results, although the A1 approach is more sensitive and accurate than the A2 in both design variables, and particularly the second. It is important to note that, for the majority of the trials, the number of iterations of the SA algorithm for the A1 and A2 approaches was roughly the same, but in certain trials, the A1 required less iterations than the A2 approach. Furthermore, if the same procedure were carried out using a single-resolution method, such as FE, and the results were subsequently decomposed to obtain the corresponding "coarse" and "fine" solutions, the following issues would arise: a) compared to the MR-FWD analyses, such models are far more time-consuming, and b) even if accuracy remained constant, the additional wavelet decomposition that is required, results in a 5–13% slower procedure, depending on the discretization, degrees of freedom and number of iterations.

2D Scenario

The inverse multiresolution method has not been fully performed in 2D cases, yet. However, as illustrated in Figure 4, the model update process could be performed in 2D cases in an analogous way as in the 1D cases. The potential of the proposed methodology for 2D cases is showcased in the wave propagation simulation of a damaged aluminum solid. The aluminum's Elastic modulus is 70 GPa, the Poisson's ratio is 0.3 and the density is 2700 kg/m^3. Its dimensions are $2m \times 2m \times 10^{-3}m$, and it is clamped at its four edges. It is excited by a horizontal 5-cycle pulse at $x=1m, y=1m$ with central frequency of 50 kHz. The analysis duration is 0.2 ms. The damage ranges from $1.35m<X<1.45m$ and $1.35m<Y<1.45m$ and is modeled as material degradation that reduces the elastic properties by 90%. For the MR model, a mesh of 200 x 200 DB3 elements is used. In Figure 6, the axial displacements (X direction) of the aluminum structure at the end of the analysis are illustrated for the coarse solution and the three fine components. Both the compression and shear wave modes are visible in the CC^0 solution. The vertical fine solution, CF^0, isolates the shear wave mode, the horizontal fine solution, FC^0, captures both wave modes, whereas the diagonal fine solution, FF^0, practically isolates the damaged region.

The coarse solution (CC^0) is now decomposed in one resolution by the DB3 wavelet, in order to produce the approximation and three detail components that are directly comparable to the three fine solutions. Those signals are shown in Figure 7, and the similarity with the fine solutions of the MR-FWD method is obvious.

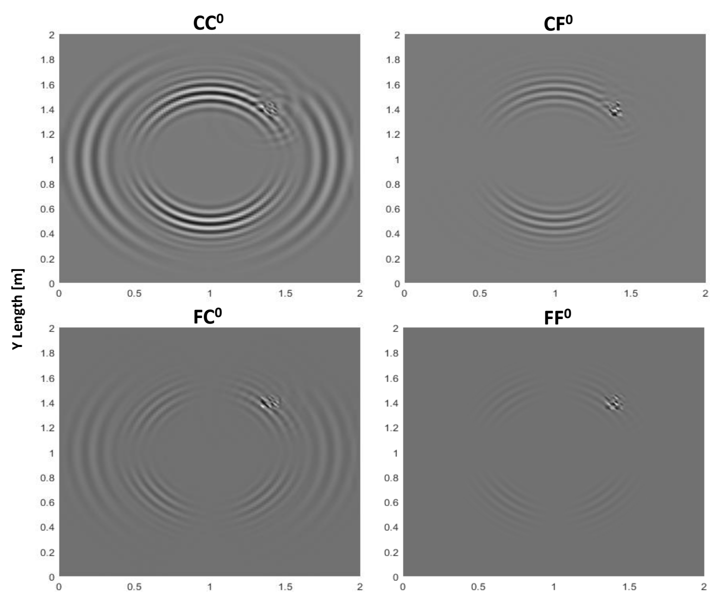

Figure 6. Axial displacement field of the coarse (CC^0), vertical fine (CF^0), horizontal fine (FC^0) and diagonal fine (FF^0) solution, at t=0.2ms.

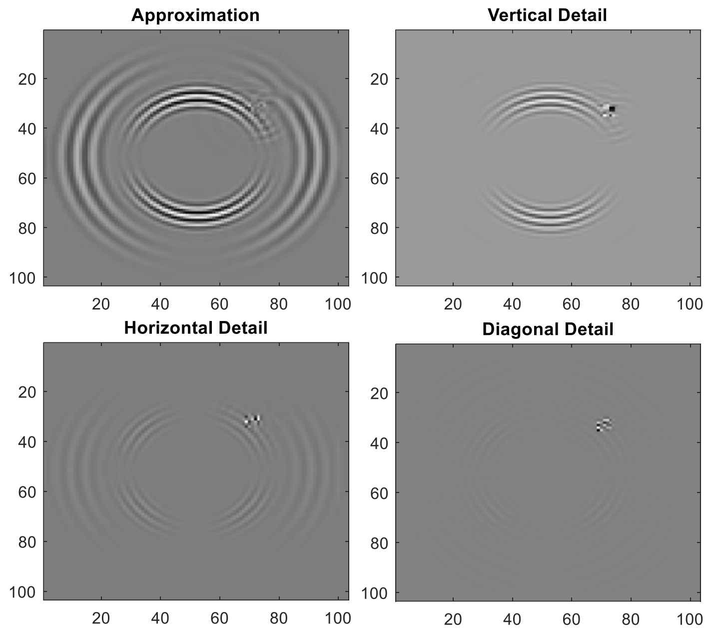

Figure 7. The four components of the wavelet decomposition of CC^0 solution that yields the approximation and three detail components.

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