Nonlinear Guided Wave Mixing for Fatigue Damages Identification in Steel Pipe

RUI ZHANG, NING LI, SHULING GAO and YANAN YUE

ABSTRACT

Nonlinear guided wave mixing techniques have been widely used for material degradation characterization in solids. In this study, a method using nonlinear guided wave mixing is proposed to identify fatigue damages, including both material degradation and subsequent fatigue crack propagation, in steel pipes. By means of theoretical analysis, two incident waves are selected to generate combined harmonics in the damaged pipes. Based on finite element (FE) simulations, the combined harmonic at difference frequency is successfully observed, and the nonlinear index, β, based on the combined harmonic at difference frequency is found to be capable of identifying both material degradation and the subsequent fatigue crack. This study provides a promising method for fatigue damages identification in the early stage using nonlinear guided wave mixing approach.

INTRODUCTION

Early detection of micro-defects in steel structures under time-varying loads is crucial for ensuring their health and integrity. As have been demonstrated in many investigations, nonlinear ultrasonics technique is a promising method for fatigue damages detection at early stage due to its high sensitivity to material microstructures, such as material degradation and fatigue crack. Compared with the bulk wave, ultrasonic guided waves can be used in large-area inspection with less transmitters [1]. Thus, ultrasonic guided wave techniques have been widely used for damage detection in pipeline industry [2].

Attempts have been made to apply nonlinear guided wave for material characterization and fatigue crack identification. The research is mainly focused on the

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second harmonic generation (SHG) and frequency mixing response, caused by the microstructural damages, such as plasticity, material degradation and fatigue microcracks. Deng [3] investigated the physical process of cumulative second-harmonic generation of Lamb-mode propagation in a solid plate. Bermes et al. [4] developed an accurate and reliable procedure to measure the second order harmonic of a Lamb wave propagating in a metallic plate, which was demonstrated by characterizing the inherent material nonlinearity of plates. Müller et al. [5] investigated the characteristics of the second harmonic generation of Lamb waves in a plate with quadratic nonlinearity, and five mode types were identified for material nonlinearity measurement. Guan et al. [6] investigated guided wave propagation and interaction with microcrack in a pipe through numerical and experimental approaches using SHG. Lee and Lu [7] identified fatigue crack in a steel joint under vibration by nonlinear guided waves based on SHG.

Although microstructural damages can lead to SHG, an apparent difficulty for the application of this technique is to isolate other sources of nonlinearity [8,9]. Therefore, it is unreliable to detect structural damages according to the second harmonic signals in practice. An alternative method to overcome the mentioned drawbacks is nonlinear guided wave mixing technique. Sun and Qu [10] proposed a nondestructive evaluation method based on one-way mixing of Lamb waves for the inspection of a large area of a plate for damage distribution via a single access point. Li et al. [11] investigated the effect of the backward combined harmonic generation induced by one-way mixing of two primary longitudinal guided waves in pipes. Jiao et al. [12] investigated nonlinear Lamb wave mixing for detection microcrack in plates. An analysis of the interaction of these waves with microcracks of various lengths and widths was performed using FE simulation. Joglekar [13] proposed an iterative use of the wavelet spectral finite element method for analyzing the phenomenon of nonlinear frequency mixing in a beam with a transverse edge-crack. However, it is noticed that nonlinear guided wave mixing techniques are not applicable for characterizing degradation and subsequent fatigue crack, which usually appear in the failed components caused by fatigue.

In this paper, we propose a method based on nonlinear guided wave mixing for material degradation and the subsequent fatigue crack identification in steel pipes. Fundamental waves are selected according to the conditions of combined harmonic generation in Section 2. In Section 3, nonlinear interaction of guided waves in steel pipes with material and crack is analyzed using FE simulation. Finally, conclusions are drawn in Section 4.

COMBINED HARMONICS FOR FATIGUE DAMAGES IDENTIFICATION

According to nonlinear guided wave mixing technique, when two incident waves propagate in the structures with material degradation, the combined harmonics at their sum and difference frequencies appear under the criteria of synchronism and nonzero power flux [14].

$$k_n = k_a \pm k_b \tag{1}$$

$$f_n^{surf} + f_n^{vol} \neq 0 \tag{2}$$

where k_n, k_a and k_b are, respectively, the wavenumber of the combined harmonic, the wavenumbers of the fundamental waves; f_n^{surf} and f_n^{vol} are the power flux from the fundamental waves to the *n*th guided wave mode caused by the surface traction and volume force. The classical nonlinear index β (Eq. 3) is commonly employed to identify and quantify the material nonlinearity.

$$\beta = \frac{A_{fn}}{A_{fa} \times A_{fb}} \tag{3}$$

where A_{fn} is the amplitude of the combined harmonic at sum or difference frequency; A_{fa} and A_{fb} are the amplitudes of the incident waves.

For fatigue crack identification based on the nonlinear response of guided wave mixing, there is no need for two incident waves meeting the demands of synchronism and nonzero power flux (previously, one incident wave initiates an 'open-and-close' behavior of the crack, whereas the other one propagates though the crack when it is closed, which leads to the appearance of combined harmonics). Moreover, the amplitudes of the fundamental waves in wave mixing for fatigue crack identification are lower than those in wave mixing for characterizing material degradation [15–17], which is a way to distinguish between material degradation and fatigue crack. Therefore, nonlinear guided wave mixing can be used in fatigue damages identification, including material degradation and subsequent fatigue crack.

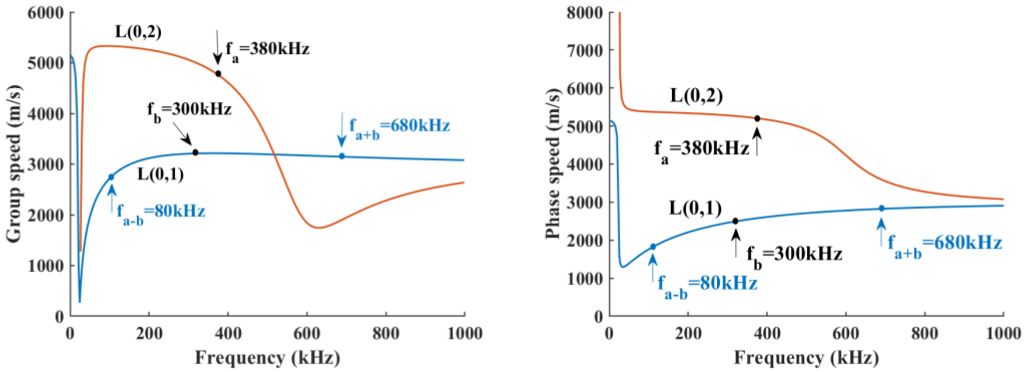
In this paper, when the material degradation and fatigue crack is detected, it is necessary for two incident waves to satisfy the requirements of synchronism and nonzero power flux. The excitation signals in this study are axisymmetric longitudinal modes, which always satisfy the criteria of nonzero power flux [18]. According to the dispersion curves in Fig. 1 and the condition of synchronism, the excitation waves are16-cycle L(0, 2) mode at 380kHz and 20-cycle L(0, 1) mode at 300kHz, named as u_1 and u_2, respectively. As summarized in Table I, only the combined harmonic at difference-frequency satisfies the synchronism condition. Thus, the nonlinear index defined as

$$\beta = \frac{A_{f1-f2}}{A_{f1} \times A_{f2}} \tag{4}$$

where A_{f1-f2} is the amplitude of combined harmonic at difference frequency; A_{f1} and A_{f2} are the amplitudes of the fundamental waves.

TABLE I. FEATURES OF THE FUNDAMENTAL WAVES AND COMBINED HARMONICS

	u_1	u_2	$u_1 + u_2$	$u_1 - u_2$
Frequency (kHz)	380	300	680	80
Group velocity (m/s)	4732	3206	3148	2506
Phase velocity (m/s)	5199	2458	2819	1626
Wave number (1/m)	459.01 (k_1)	766.48 (k_2)	1514.86 (k_+)	308.98 (k_-)
Δk(1/m)	-	-	-289.37	1.51



(a) group velocity (b) phase velocity

Figure 1. Dispersion curve of steel pipe with 73 mm diameter and 4 mm thickness.

To obtain the combined harmonic at difference-frequency, two fundamental waves are excited at the left end of the pipe one after another with the specific time delay so as to accomplish a total wave mixing at a distance of 450 mm from the left end. Two excitation signals used in the simulation study are shown in the Fig. 2. In Fig. 2(b), the horizontal segment before the waveform represents the time delay.

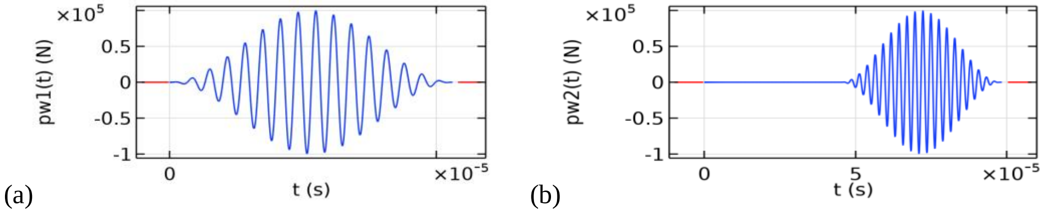


Figure 2. Excitation signals (a) L(0,1) mode, (b) L(0,2) mode.

The nonlinear strain energy function W can take material nonlinearity into account, as the inclusion of the third order terms. Based on the study of Murnaghan [19], W can be written in the form

$$W(E)=\frac{1}{2}(\lambda+2\mu)i_1^2+\frac{1}{3}(l+2m)i_3^2-2\mu i_2-2mi_1i_2+ni_3 \tag{5}$$

where λ, μ are the Lame elastic constants; l, m, n are the third order elastic constants; $i_1, i_2, \ i_3$ are the principal invariants of E (Green-Lagrange strain tensor). The material parameters of the steel pipe are listed in Table II [20].

TABLE II. MATERIAL PARAMETERS OF THE STEEL PIPE [20].

$\boldsymbol{\rho(kg/m^3)}$	$\boldsymbol{E(GPa)}$	$\boldsymbol{\lambda(GPa)}$	$\boldsymbol{\mu(GPa)}$	$\boldsymbol{l(GPa)}$	$\boldsymbol{m(GPa)}$	$\boldsymbol{n(GPa)}$
7829	207	94	82	-795	-318	-1105

FINITE ELEMENT SIMULATION

As the incident waves are axisymmetric longitudinal guided wave modes, 2D axisymmetric models of the steel pipe with a length of 800 mm, an outer diameter of 73 mm and a thickness of 4 mm can be made in the commercial software COMSOL Multiphysics for fatigue damages identification. Fundamental waves are excited at the left end of the pipe, and the low-reflecting boundary condition is added to the right end to reduce the influence of reflected waves.

With regard to the simulation model for characterizing material degradation, the third order elastic constants are modified to introduce material degradation (Fig. 3). To identify fatigue crack that occur after the material degradation, a simulation model with the material nonlinearity and a contact crack is found, as illustrated in Fig. 4.

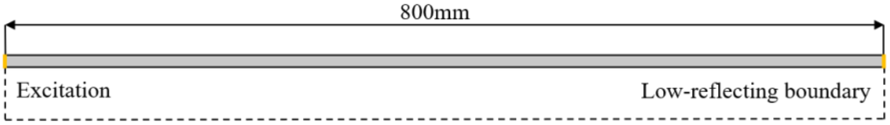


Figure 3. Simulation model for characterizing material degradation.

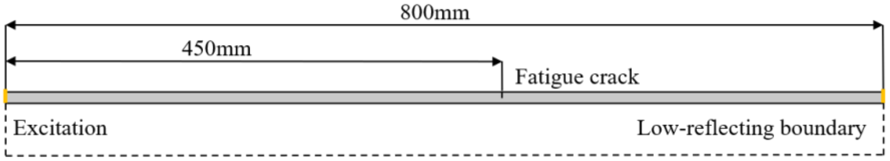


Figure 4. Simulation model for fatigue crack identification.

The maximum element size (ΔI) and time step (Δt) are adopted as follows [21]

$$\Delta I = \frac{\lambda_{min}}{20} \text{ and } \Delta t = \frac{1}{20 f_{max}} \tag{6}$$

where λ_{min} and f_{max} are the minimum wavelength and the maximum frequency of the incident waves used for combined harmonics generation. Considering the combined harmonic at sum frequency (680kHz), the determined values for ΔI and Δt are 0.2 mm and 5e-8 s, respectively. Mapped meshes are utilized in the 2D axisymmetric models (Fig. 5) to ensure accuracy as well as computational efficiency.

Figure 5. An example of mapped meshes.

Mode verification

To obtain the combined harmonics, the time-domain generated signals of L(0, 1) and L(0, 2) modes should be checked at first. Adding radial excitation at the left end of the pipe yields a single L(0, 1) mode, while axial excitation yields a single L(0, 2) mode [17], as shown in Fig. 6.

Time-domain signals, as shown in Fig. 7, are received by the probes at the distance of 300mm from the left end of the pipe. The first waveform is the direct wave, and the second one is the reflection signal from the right end of the pipe. In Fig. 7, it can be observed that only the waveform of L(0, 2) mode broadens slightly during propagation due to the dispersion. The simulated wave velocity of the L(0, 1) mode is determined to be 3194.38 m/s, which represents an error of 0.36% compared to the theoretical value of 3206 m/s; the simulated wave velocity of the L(0, 2) mode is calculated to be 4679.46 m/s, similar to the theoretical value of 4732 m/s. These results confirm that the L(0, 1) mode can be excited by the radial excitation, while the L(0, 2) mode can be excited by the axial excitation.

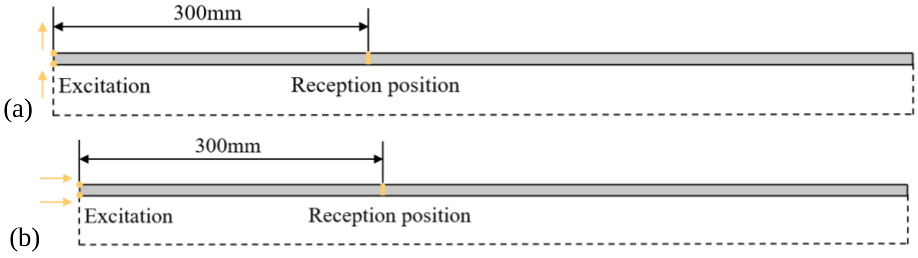
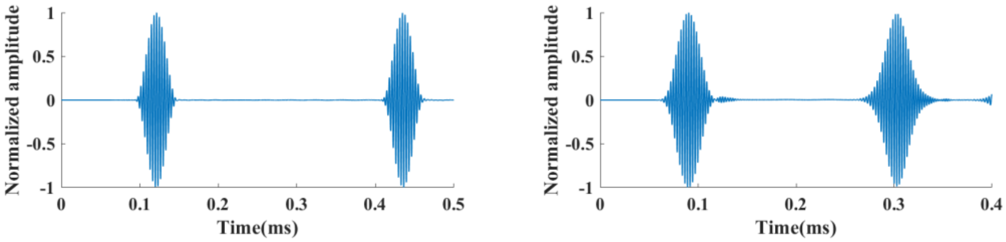


Figure 6. The excitation methods for the single mode generation (a) L (0,1) mode, (b) L (0,2) mode.



(a)

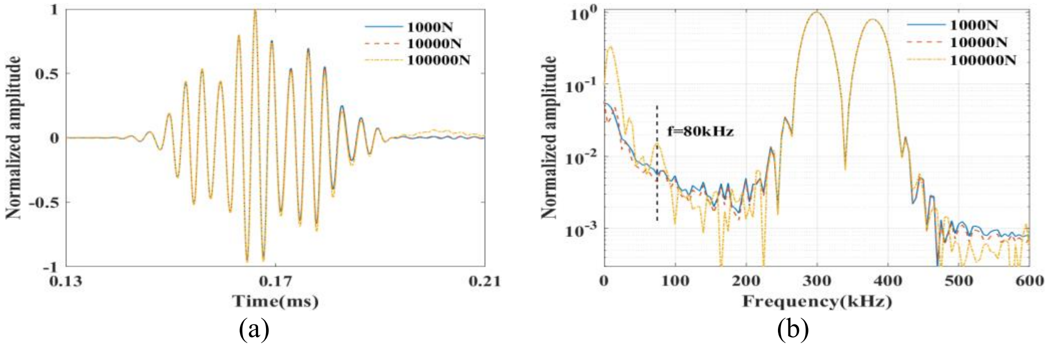
(b)

Figure 7. Time-domain signals (a) L(0, 1) mode, (b) L(0, 2) mode.

Identification of material degradation

Since harmonic generation in the component with material nonlinearity is associated with finite amplitude waves having a sufficiently large displacement gradient [17], the influence of the amplitudes of the incident waves on the wave mixing effect is investigated, firstly. Two incident waves have the same amplitude of 1000N, 10000N and 100000N in three different cases. Fig. 8 shows the normalized waveforms and frequency spectra obtained at the wave mixing center (450 mm from the left end of the pipe). It can be observed that there is little difference in the time-domain signals for different amplitude excitations, while a clear combined harmonic at difference frequency (80kHz) can only be observed at an amplitude of 100000 N. Therefore, the amplitudes of two fundamental waves are set to 100000 N in subsequent analyses to achieve the desired material degradation identification.

To identify the material degradation in the steel pipe, three cases are designed by magnifying the third-order elastic constants defined by Murnaghan function (Eq. 5), as listed in Table III. Case 1 represents the health state of the pipe, while case 2 and 3 represent the material degradation states of the pipe. Fig. 9 gives the frequency-domain signals in all cases. Combined harmonic at difference frequency can also be observed in the undamaged state, which is attributed to the intrinsic physical nonlinearity of the material. According to Eq. 4, the nonlinear index β is calculated, and the results are shown in Table IV. It is evident that the damage index β is well correlated with the degree of material degradation, which can be used to characterize the degradation state of the material. Fig. 10 illustrates 3D propagation of the incident waves.



(a) (b)

Figure 8. (a) Time-domain mixing signals, (b) frequency spectra.

TABLE III. THE THIRD ORDER ELASTIC CONSTANTS IN THREE SITUATIONS (GPA).

	l	m	n
case 1	-795	-318	-1105
case 2	-1590	-636	-2210
case 3	-3180	-1272	-4420

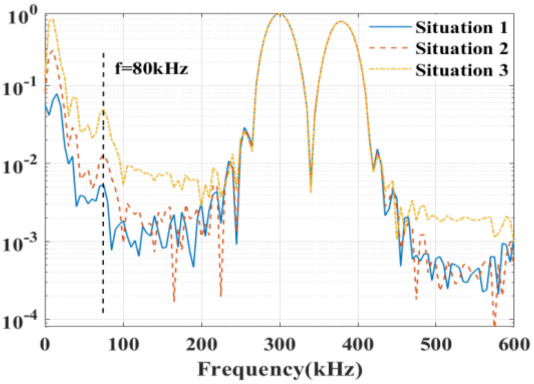


Figure 9. Frequency spectra in three situations.

TABLE IV. NONLINEAR INDEX β IN THREE MATERIAL DEGRADATION SITUATIONS.

	Situation 1	**Situation 2**	**Situation 3**
Index β	0.008	0.020	0.074

Identification of fatigue crack

As discussed in the previous section, material degradation can only be identified when amplitudes of two fundamental waves are sufficiently large. Therefore, the combined harmonic can only be attributed to the presence of the fatigue crack when using incident waves with amplitudes of 10 N. Hereby, the amplitudes of the incident waves are set to 10 N, while all other signal parameters are kept consistent with those used for material degradation characterization.

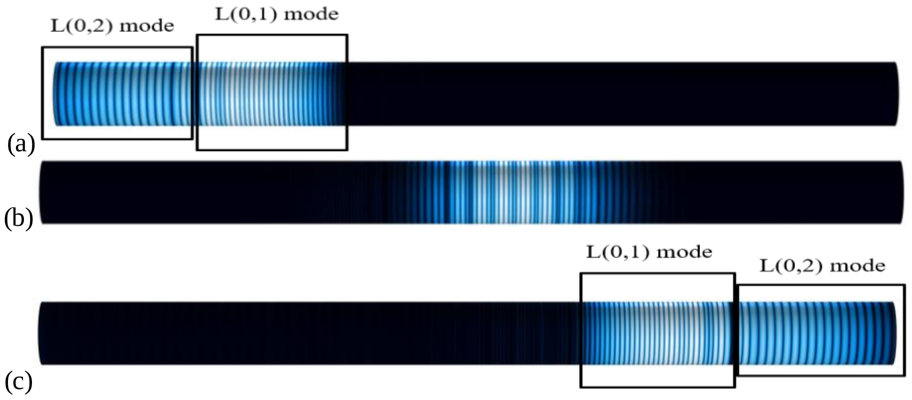
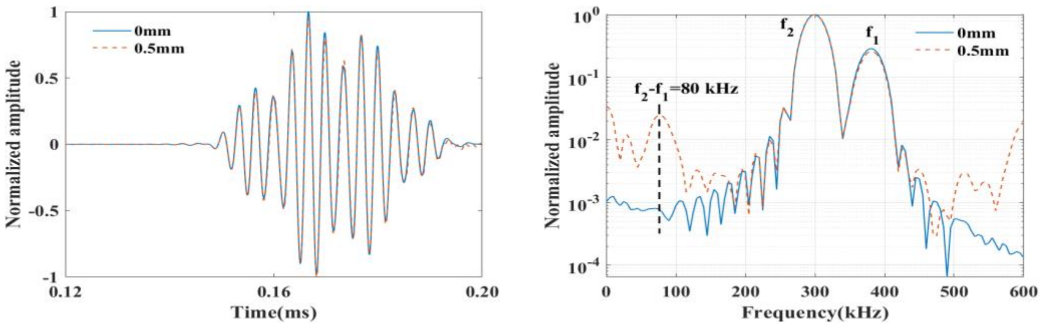


Figure 10. The 3D propagation of the incident waves (a) before wave mixing, (b) during wave. mixing and (c) after wave mixing.

To demonstrate that only the presence of the fatigue crack can result in combined harmonic at difference frequency when low-amplitude fundamental waves are excited, the simulation model with material degradation (case 3 in Table III) and the simulation model with both material degradation and 0.5 mm fatigue crack are investigated at first, the results are shown in Fig. 11. It can be observed that there is little difference in the time-domain signals for two cases, while a clear combined harmonic at difference frequency (80kHz) can only be observed in the model with both material degradation and 0.5 mm fatigue crack. Moreover, the nonlinear index β for charactering material degradation can also be used for fatigue crack identification according to Fig. 11(b).

Next, three fatigue crack cases, as listed in Table V, are established to identify the fatigue crack that occurs after the material degradation. The frequency spectra are given in Fig. 12, which indicates that the amplitude of the combined harmonic at difference frequency increases with the longer crack length. Following the Fig. 12, the nonlinear index β is calculated, and the results are provided in Table VI. It is obvious that β is well correlated with the degree of damage.



(a) (b)

Figure 11. (a) Time-domain mixing signals, (b) frequency spectra.

TABLE V. CASES FOR FATIGUE CRACK IDENTIFICATION.

	case 1	case 2	case 3
Crack length	0.5 mm	1.0 mm	1.5 mm

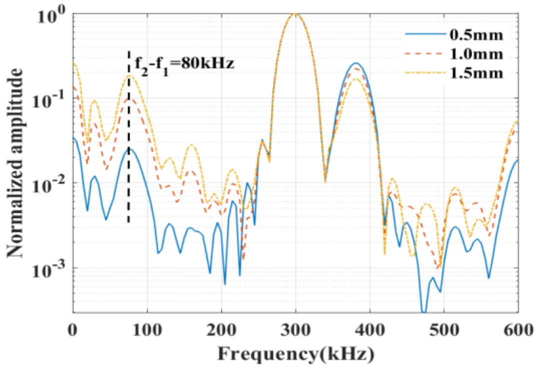


Figure 12. Frequency spectra in three situations.

TABLE VI. NONLINEAR INDEX β UNDER THREE FATIGUE CRACK CONDITIONS.

	0.5mm	1.0mm	1.5mm
Index β	0.096	0.454	1.090

CONCLUSIONS

A method using nonlinear guided wave mixing is proposed to identify material degradation as well as subsequent fatigue crack propagation in steel pipes. Through theoretical analysis, L(0,1) and L(0,2) modes that satisfy the requirements for combined harmonics generation are selected for wave mixing. Mode verification is conducted to ensure proper excitation of the fundamental waves firstly, followed by the material degradation and fatigue crack identification. These simulations show that fundamental waves can be excited correctly with the specific excitation ways. In addition, for the steel pipe with only material degradation, combined harmonic at difference frequency is sensitive to the damage with the high-amplitude waves excitation; for the steel pipe with the fatigue crack and material degradation, fatigue crack can be identified separately by reducing the amplitudes of two incident waves, and the combined harmonic at difference frequency is also sensitive to the presence of fatigue crack in pipes. Furthermore, the nonlinearity index β based on combined harmonic at difference frequency is well correlated with the degree of fatigue damages at early stage.

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