Exploring Compressed Sensing Approaches for ToA Estimation in Thin Composite Plates

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ABSTRACT

Guided Wave-based Structural Health Monitoring (GWSHM) systems make a promising technique for monitoring and evaluating the integrity of many industrial structures like pipelines, aircraft, and civil structures. Composite structures used in aircraft must continue in flight-safe conditions when subjected to various levels of damage and operating conditions, such as stress, load, temperature, humidity, during manufacturing and operational usage. Even though the GWSHM system has the capability to observe the structure integrity continuously, it requires a significantly high sampling rates that become a burden while recording and transmitting the data for continuous monitoring. Recording data with a high sampling rate results in high energy consumption during transmitting/uploading, resulting in a bottleneck issue for modern embedded SHM systems that may be battery-operated and/or have a wireless connection.

One possible solution is applying compression during the acquisition stage, thereby keeping only the relevant information. This article proposes a compressed sensing-based framework for estimating the time of arrival (ToA), which is a key parameter in signal recovery, damage detection, localization, and imaging applications. In this framework, the ToA is estimated by solving a sparse deconvolution problem with Orthogonal Matching Pursuit (OMP), investigating different sparsity-providing bases such as Discrete Fourier Transform (DFT), Discrete Wavelet Transform (DWT), and Discrete Cosine Transform (DCT). These bases can be combined with subsampling strategies for compression. For recovery, several grid-free techniques such as the Atomic Norm Minimization (ANM) and parametric high-resolution ToA estimators such as Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT), are explored.

The methods are tested on synthetically generated data as well as experimental measurements on composite plates. The results are compared to traditional methods for ToA estimation using the maximum peak of the envelope of the Hilbert transform of the received signal, providing a complete framework for resource-efficient ToA estimation in SHM applications.

INTRODUCTION

Guided Wave-based Structural Health Monitoring (GWSHM) systems have the capability to continuously monitor the structure integrity, resulting in generating not only large amounts of data, but also each recorded measurement has a large size due to high sampling rate. This data must either be stored, requiring substantial storage and computational resources that may not be available, or transmitted, needing a stable, high-bandwidth communication link, which is often impractical [1],[2].

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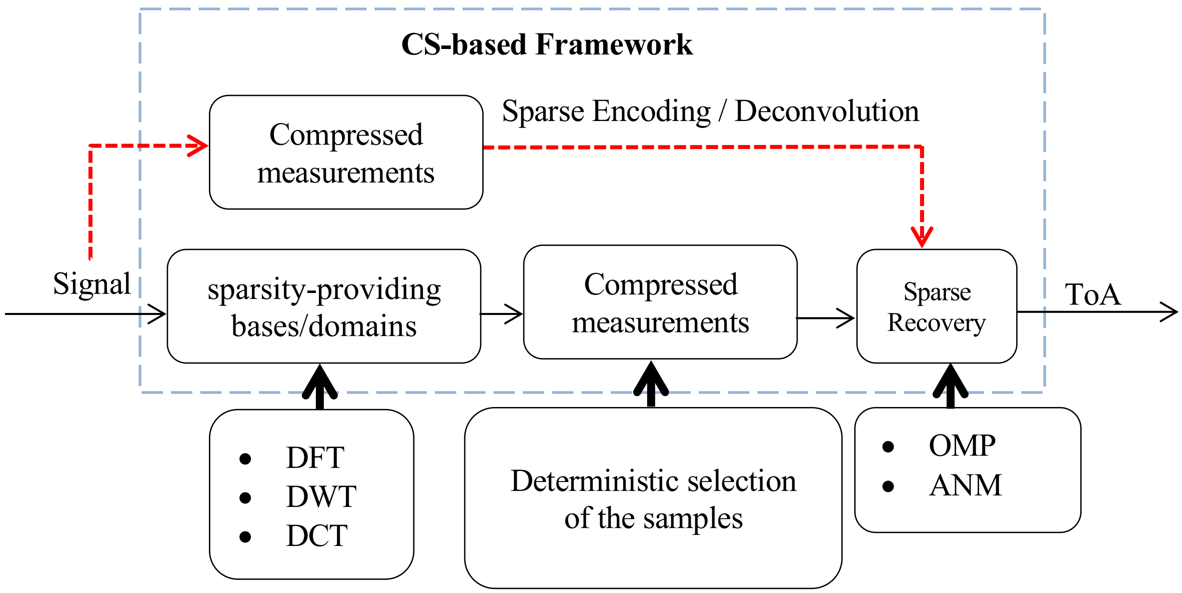

Figure 1. Flowchart illustrating the ToA estimation from compressed data.

Both approaches consume significant energy, a critical concern for embedded SHM systems on limited energy budgets. This results in a significant storage problem that can affect continuous monitoring capabilities.

One of the possible solutions to overcome the high transfer size of the recorded measurements is Compressed Sensing (CS). It can be applied by exploiting signal sparsity, enabling reconstruction from sub-Nyquist measurements [3], [4]. The recorded signals are sparse in bases like Discrete Fourier Transform (DFT), Discrete Wavelet Transform (DWT), and Discrete Cosine Transform (DCT) [5], allowing CS to envision data acquisition architectures that tailor how the data can be captured and compressed without sacrificing data fidelity.

One of the most useful extracted features within the GWSHM recorded measurement is the time of arrival (ToA). The ToA reflects the wave propagation speed within the material, which is highly sensitive to variations in material properties. In a dispersive medium, these variations can be influenced by factors such as frequency-dependent dispersion and temperature. Various signal processing techniques can be used to estimate the ToA, such as detecting the maximum peak of the envelope obtained by the Hilbert transform [6].

In this paper, besides sparse deconvolution for ToA estimation [7], [8], ToA estimation techniques are designed for compressed or subsampled signals from a CS, discrete/continuous sparse signal recovery and estimation perspective.

Figure 1 demonstrates the possibilities to estimate the ToA after the signal is compressed. The compression can be applied after the full data is acquired by storing the data in a sparse coefficient through transformation. Also, the compression can be integrated directly during the acquisition phase, where only a limited number of strategically selected measurements are collected. This can be employed by the fact that prior knowledge of the signal is known. The choice between post-acquisition compression and direct compressed acquisition is highly dependent on the system's design and computational constraints.

DATA ACQUISITION AND COMPRESSION

Compressed Sensing

Compressed Sensing enables simultaneous acquisition and compression of signals, significantly reducing data volume without compromising signal fidelity. In this work, a CS-based framework tailored to ToA estimation is proposed. The framework is designed to simulate the sensing process such that compression is integrated directly at the acquisition phase. The CS technique relies heavily on prior knowledge of the received signal. In this case study, this includes dedicated studies of the propagation characteristics of ultrasonic guided waves in an undamaged composite plate, with future extensions planned for damaged conditions. To exploit sparsity in the signal representation, different domains such as DFT, DWT, and DCT are investigated. For measurement compression, a selection matrix that performs subsampling by selecting frequency-domain samples centered around the center frequency is designed, where those coefficients are previously determined to have significant energy in the spectrum. With these considerations, the compressed sensing model can be expressed as:

$$\boldsymbol{y} = \boldsymbol{\Phi\Psi A x} + \boldsymbol{n} \;\in \mathbb{C}^{N_{\mathrm{F}}} \tag{1}$$

Where vector y denotes compressed measurement of a single channel, $\boldsymbol{\Phi}$ represents a compression matrix that retains only N_{F} coefficients from the spectrum, focusing on the region around the center frequency of the sparsity-providing bases $\boldsymbol{\Psi}$. $\boldsymbol{A}$ dedicates the forward model, e.g., a pulse shape matrix that models the expected pulse shape based on the transducer response, $\boldsymbol{x}$ represents the sparse coefficient vector, which in our case contains the ToA and amplitude information, and $\boldsymbol{n}$ is additive white Gaussian noise.

Forward Modeling and Parameter Estimation

The received signal model can be expressed as:

$$\boldsymbol{s}(\boldsymbol{t}) = \boldsymbol{g}(\boldsymbol{t}) * \;\boldsymbol{p}\,(\mathrm{t}-\tau) + \boldsymbol{n} \tag{2}$$

Where $\boldsymbol{s}(\boldsymbol{t})$ is the received signal, $\boldsymbol{p}(\boldsymbol{t})$ represents the impulse response, and τ is the time delay. The $\boldsymbol{g}(\boldsymbol{t})$ is a gain factor that, in a more complete model, can account for dispersion and temperature effects. However, for this study, $\boldsymbol{g}(\boldsymbol{t})$ is neglected. The convolution operator $*$ indicates that the received signal is the result of the impulse response being filtered to match the actual signal model, where the effects of dispersion and temperature are present.

The impulse response is often not exactly known [8]. Instead, a parameterized pulse-echo model is adopted to simulate the transducer response following the approach proposed in [8]. This model allows for the estimation of key pulse parameters, including center frequency, phase, and amplitude. To simulate the system response, the A matrix is constructed as a Toeplitz matrix. Each column of this matrix represents a time-shifted version of the estimated pulse shape, effectively capturing the system's temporal characteristics.

Inverse Problem

Solving Equation (1) as an inverse problem involves estimating $\boldsymbol{x}$, i.e., the sparse representation of the signal that best maps the compressed measurement $\boldsymbol{y}$ with a prior knowledge of $\boldsymbol{A}$. Depending on the nature of the problem and the desired balance between accuracy and computational efficiency, this can be achieved using various sparse deconvolution techniques, including convex optimization, greedy algorithms, and continuous sparse recovery. In this paper, we compare different deconvolution approaches, which are discussed in the next section.

SIGNAL RECOVERY AND TOA ESTIMATION

Sparse Signal Recovery

In this study, the direct-path ToA is defined as the maximum peak in the envelope of the received signal. For a linear sensor array, each received signal can be modeled as a time-shifted replica of the signal recorded by the nearest sensor to the transmitter. This model can be mathematically represented using a Toeplitz matrix. Then, ToA can be estimated by applying sparse deconvolution techniques to sparsely encode the received signal [9]. In this article, discrete sparse deconvolution techniques, such as OMP, and the continuous ones, such as ANM, are compared.

ORTHOGONAL MATCHING PURSUIT

Due to the simplicity of implementation, orthogonal matching pursuit (OMP), a greedy algorithm that decomposes each received signal into a sparse vector by iteratively selecting dictionary atoms maximally correlated with the current residual and updating the coefficients with the orthogonal projection, is used. The resulting sparse vectors typically contain a single dominant non-zero coefficient corresponding to the ToA expressed as:

$$\boldsymbol{x} = \text{argmin}_{\boldsymbol{x}} \|\boldsymbol{\Phi\Psi A x} - \boldsymbol{y}\|_2^2 \quad s.t \quad \|\boldsymbol{x}\|_0 \leq k \tag{2}$$

where k denotes the sparsity level, and $\|.\|_0$ represents the $l0-norm$.

ATOMIC NORM MINIMIZATION

In this work, we also explore atomic norm minimization (ANM) as a grid-free sparse recovery technique to estimate the ToA. Unlike traditional sparse recovery techniques such as OMP, which rely on discretized time-delay grids, ANM reformulates the sparse recovery problem as a semidefinite program (SDP) defined over a continuous parameter space. This allows more accurate ToA estimation, avoiding the discretization in grid-based methods [10]. For this setup, the spectrum of the received signal is normalized by the frequency response of the pulse shape to isolate the delay-dependent complex exponentials. Then, a data-fitting constraint is defined, and the ANM problem is solved using convex optimization using CVXPY [11]. The solution yields complex coefficients whose phases correspond to time delays. The theoretical foundation of this method is comprehensively detailed in [10].

Estimation Of Signal Parameters Via Rotational Invariance Techniques

Estimation of signal parameters via rotational invariance techniques (ESPRIT) can be used for ToA estimation by modeling the received signal as a sum of delayed complex exponentials of a known pulse shape [12]. This technique follows the same setup as described in ANM. This yields a sum of weighted exponentials whose phases encode the time delays. A Hankel matrix can then be constructed to impose shift-invariance, and the dominant signal subspace is extracted using the Singular Value Decomposition (SVD). Two selection matrices extract overlapping subarrays of this subspace to enforce rotational invariance; A transformation matrix is estimated using least squares, whose eigenvalues encode the phase shifts caused by time delays, and unwrapping the angles of these eigenvalues corresponds to the unknown ToAs. This method is particularly effective when the signal comprises a sparse set of delayed pulses and sufficient samples are available to capture the shift-invariant structure.

EXPERIMENTAL SETUP

To validate the proposed model, experiments were conducted on an undamaged quasi-isotropic AS4/8552 Carbon Fiber Reinforced Plastic (CFRP) composite plate with dimensions of 260 × 460 mm and a thickness of 1.84 mm, prepared by the Universidad Politécnica de Madrid (UPM) and tested at Université Gustave Eiffel. The layup configuration followed a symmetric stacking sequence of [+45/−45/0/90/0/0/90/0/−45/+45]. The experiments utilized the Ondulys probe shown in Figure 1, which consists of 13 Acsys P-wave S1803 transducers that were dry-coupled to the plate. A total of 40 signals were linearly recorded along the surface of the panel, with uniform spacing between the transducers. Data acquisition was performed at a rate of 2 Msps. Active interrogation was carried out every 5 minutes, ensuring continuous monitoring of the structural response over time. Each recorded signal contained 2000 samples, obtained by averaging 1024 acquisitions to enhance signal quality. The transmitted waveform was a Ricker pulse (burst of 1.5 cycles) centered at 100 kHz.

Figure 2. The test specimen setup

RESULTS AND DISCUSSION

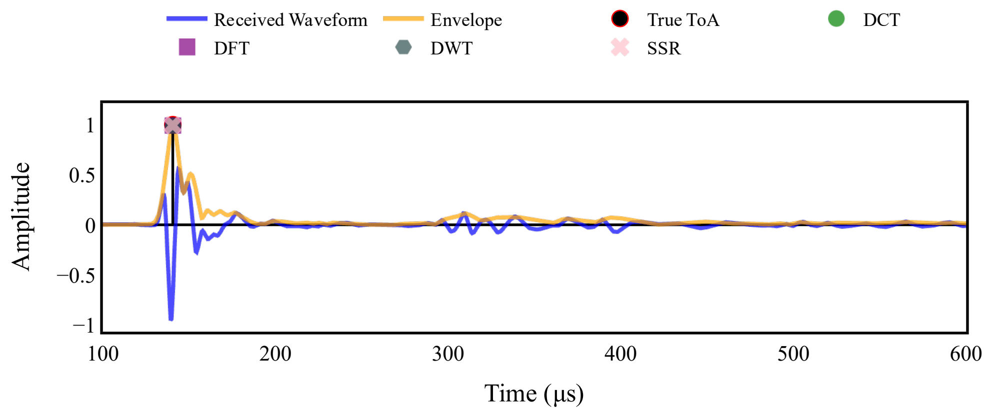

Figure 3 Signal recorder at an offset of 25 mm, with ToA estimated from compressed measurements.

Figure 3 presents the ToA estimates derived from compressed measurements, where only 300 coefficients around the center frequency were retained from the spectrum (compared to 2000 samples in the original signals). The figure compares ToA estimation performance using three different sparse-providing bases: DCT, DFT, and DWT. Under this setup, all bases perform stably and accurately estimate the ToA.

However, to evaluate the effect of further compression on ToA estimation accuracy, the coefficients were gradually reduced one by one, and the resulting distortion is shown in Figure 4. The results indicate that the DWT bases achieve 7-fold compression without compromising ToA accuracy, while the DFT and DCT bases provide 13-fold compression with equal precision. In addition, for DCT/DFT, there is a graceful degradation, meaning higher compression can still be used with a moderate degradation of the ToA estimation.

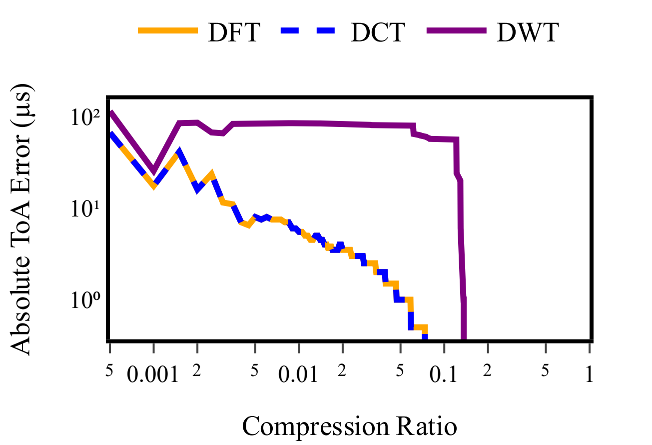

Figure 4. Absolute ToA error for different compression levels where the ground truth ToA is 140.5 μs.

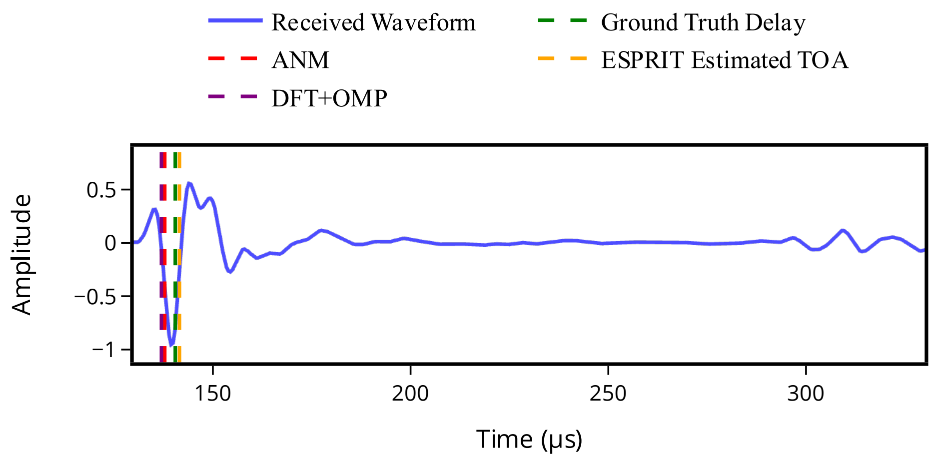

Figure 5. ToA estimated from compressed measurements using grid-free techniques.

Figure 5 compares two grid-free techniques for ToA estimation (ANM and ESPRIT) against the traditional discrete OMP method. All three techniques were evaluated using only 34 coefficients around the center frequency. The results show that the latter provides the closest ToA estimation to the ground truth, achieving 57-fold compression.

In contrast, Figure 6 focuses on the accuracy of signal reconstruction, each method is applied using its optimal compression level based on the best ToA estimation. In this scenario, ANM outperforms the other methods, achieving the lowest Root Mean Square Error (RMSE) for signal reconstruction. Notably, for ANM and ESPRIT, reconstruction accuracy can be further improved using model order selection (MOS) approaches, such as the akaike information criterion (AIC) [13]. These MOS techniques determine the optimal sparsity level required for accurate signal reconstruction.

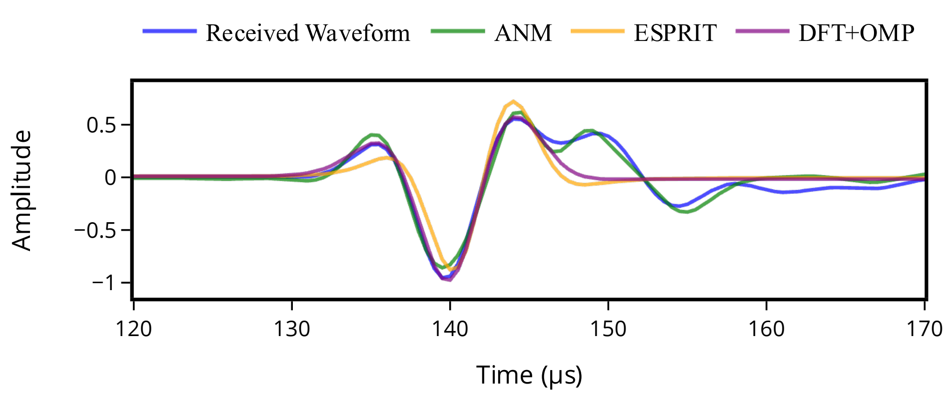

Figure 6. ToA estimated from compressed measurements using grid-free techniques.

CONCLUSIONS

In this study, the performance of various sparse-providing bases and grid-free techniques for ToA estimation from compressed measurements is evaluated. The analysis demonstrates that all tested bases (DCT, DFT, and DWT) maintain accurate ToA estimation, achieving 7-fold compression. Among the grid-free techniques, ESPRIT demonstrates the highest accuracy in ToA estimation, achieving 80-fold compression, while ANM provides the lowest Root Mean Square Error (RMSE) for signal reconstruction at 58-fold compression.

Additional analysis has shown that ESPRIT and ANM are found to be sensitive to certain conditions, including the requirement for accurate prior knowledge of model order and sampling strategies that preserve ToA information under varying dispersion and temperature conditions, necessitating an extended framework that accounts for these effects. Otherwise, they can fail dramatically, producing erroneous ToA estimates.

FUTURE WORK

Future work can aim to extend the current framework to incorporate dispersion modeling and environmental and operational conditions (EOC) compensation to enhance the accuracy of ToA estimation in more realistic scenarios. Another critical direction is evaluating the framework's applicability to damaged composite plates. Adapting the approach to such complex and real conditions would significantly broaden its usability in real-world applications.

ACKNOWLEDGMENT

This research work was performed within the European project USES2 - USES of Novel Ultrasonic and Seismic Embedded Sensors for the non-destructive evaluation and structural health monitoring of infrastructure and human-built objects (https://www.uses2.eu/), which has received funding from the European Union's Horizon 2020 research and innovation program under the Marie Skłodowska-Curie Grant Agreement No. 101072599.

REFERENCES

[1] Y. Lu, R. Demirli, and J. Saniie, "NDE applications of compressed sensing, signal decomposition and echo estimation," in *IEEE International Ultrasonics Symposium, IUS*, IEEE, 2014, pp. 1928–1931.

[2] Z. Lu, C. Yang, D. Qin, Y. Luo, and M. Momayez, "Estimating ultrasonic time-of-flight through echo signal envelope and modified Gauss Newton method," *Measurement (Lond)*, vol. 94, pp. 355–363, Dec. 2016.

[3] J. Kirchhof, S. Semper, C. W. Wagner, E. Perez, F. Romer, and G. Del Galdo, "Frequency Subsampling of Ultrasound Nondestructive Measurements: Acquisition, Reconstruction, and Performance," *IEEE Trans Ultrason Ferroelectr Freq Control*, vol. 68, no. 10, pp. 3174–3191, Oct. 2021.

[4] E. Pérez, J. Kirchhof, S. Semper, F. Krieg, and F. Römer, "Total Focusing Method with Subsampling in Space and Frequency Domain for Ultrasound NDT," *IEEE International Ultrasonics Symposium (IUS)*, p. 2644, 2019.

[5] M. Yousufi *et al.*, "Application of Compressive Sensing to Ultrasound Images: A Review," *Biomed Res Int*, vol. 2019, 2019.

[6] L. Svilainis, “Review of high resolution time of flight estimation techniques for ultrasonic signals,” Conference: NDT 2013 At: Telford Project: In-SMART (2013)., Sep. 2013.

[7] C. Xu, Z. Yang, S. Tian, and X. Chen, “Lamb wave inspection for composite laminates using a combined method of sparse reconstruction and delay-and-sum,” *Compos Struct*, vol. 223, Sep. 2019, doi: 10.1016/j.compstruct.2019.110973.

[8] F. Boßmann, G. Plonka, T. Peter, O. Nemitz, and T. Schmitte, “Sparse deconvolution methods for ultrasonic NDT : Application on TOFD and wall thickness measurements,” *J Nondestr Eval*, vol. 31, no. 3, pp. 225–244, Sep. 2012.

[9] R. Demirli and J. Saniie, “Model-based estimation of ultrasonic echoes part I: Analysis and algorithms,” *IEEE Trans Ultrason Ferroelectr Freq Control*, vol. 48, no. 3, pp. 787–802, May 2001, doi: 10.1109/58.920713.

[10] Y. Park, W. Seong, and Y. Choo, “Compressive time delay estimation off the grid,” *J Acoust Soc Am*, vol. 141, no. 6, pp. EL585–EL591, Jun. 2017, doi: 10.1121/1.4985612.

[11] S. Diamond and S. Boyd, “CVXPY: A Python-Embedded Modeling Language for Convex Optimization,” *Journal of Machine Learning Research*, vol. 17, pp. 1–5, 2016.

[12] R. Roy and T. Kailath, “ESPRIT-Estimation of Signal Parameters Via Rotational Invariance Techniques,” *IEEE Trans Acoust*, vol. 37, no. 7, 1989.

[13] Petre. Stoica and R. Moses, *Spectral analysis of signals*. Pearson/Prentice Hall, 2005.