

# **A New Comparison Index of Estimation Accuracy for Wave Load-Based DBM Superposition Method on Marine Structures**

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## **ABSTRACT**

The hull stress monitoring (HSM) system, which is the early form of sensor-based structure health monitoring (SHM) in the marine industry, has the limitation that it can only monitor the response at the location where the sensor, long-based strain gauge (LBSG), is attached. To overcome this limitation, the wave load-based distortion basis mode (DBM) superposition method was proposed. This method requires numerous case studies to find the optimal LBSG configuration with best estimation accuracy. To facilitate those studies, efficient way of comparison on the estimation accuracy between cases is necessary. In this study, a new index that can consider both the amplitude and phase shift of the response is introduced. With this index, study with over 150,000 cases are performed for a semi-submersible structure, and the estimation accuracy is compared according to the total number of LBSGs.

## **INTRODUCTION**

The specific goal of structure health monitoring (SHM) for the marine structures is to estimate global sectional loads and local stresses, and continuously monitor whether they meet design criteria for strength and fatigue. The estimation can be implemented with or without structural measurement sensors, but the classification societies certify sensor-based SHM to be a higher tier [1-2]. As structural measurement sensors, long-based strain gauge (LBSG) is normally adopted to minimize the effects of local disturbances or structural vibrations.

This paper briefly introduces the hull stress monitoring (HSM) system with 4 LBSGs, which is the early form of SHM system in the marine industry, and the wave-load based distortion basis mode (DBM) superposition method to overcome limitations of HSM system. Furthermore, an idea for more effective use of the DBM superposition method is presented, that is, an index for judging the superiority relationship between numerous cases. Finally, an actual case study on a semi-submersible structure is performed by applying the new index, and compares the prediction performance according to the number of LBSGs.

## HULL STRESS MONITORING SYSTEMS

As several accidents of prominent bulk carriers occurred in the 1980s, International Maritime Organization (IMO) recommended in 1994 that ships be equipped with HSM systems [3]. Classification societies have also published technical standards and requirements for the use of HSM systems [4-6].

Due to the slender and long shape of ship structures, the bending moment, especially the vertical bending moment (VBM) acts as the most dominant global sectional load, and various design standards/rules present the limiting values against VBM for design. VBM is maximum amidships for the lowest order vibration mode, and maximum at 1/4 and 3/4 of the ship length for the second order vibration mode. For this reason, the HSM system is standardized to place two LBSGs amidships, at port and starboard side symmetrically, and ones each at 1/4 and 3/4 of the ship length. This 1/2/1 LBSG configuration has been adopted by various commercial packages, and by classification societies as a recommendation [7]. Figure 1 shows the sensor configuration of HSM system recommended by American Bureau of Shipping (ABS) for common ships.

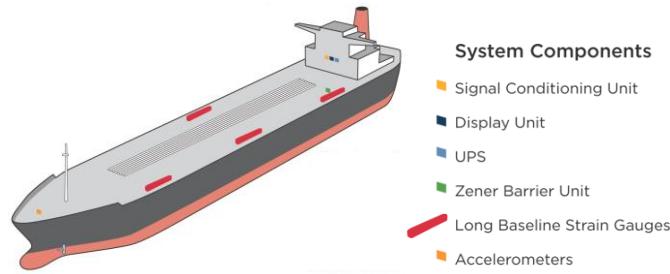


Figure 1. Recommend sensor configuration of HSM system for common commercial ships by ABS

Assuming that bending moments are the only global loads on section, VBM ( $M_V$ ) can be calculated amidships by simple beam theory, as follows:

$$M_V = E \cdot I_A / z_c \cdot (\varepsilon_p + \varepsilon_s) / 2 \quad (1)$$

where  $E$  is Young's modulus,  $I_A$  is second moment of area with respect to the neutral axis,  $z_c$  is vertical distance between LBSG and neutral axis,  $\varepsilon_p$  and  $\varepsilon_s$  is strain measured from LBSGs at port and starboard side respectively. At other locations where only one LBSG is placed, VBM can be calculated by applying same rate between strain and horizontal distance as midship.

## SUPERPOSITION OF DISTORTION BASIS MODES

### Definition

The HSM system has a limitation in that it is only capable to monitor the (longitudinal) locations where the LBSGs are placed. The superposition of distortion basis modes (DBM) method has been applied as a method to predict the global loads and local stresses at arbitrary locations where the LBSGs are not placed. The method is to approximate the distortion of overall structure as a linear combination of distortions

by limited number of modes. Then, the strain at LBSGs and the global load or stress at any location can be related as following:

$$F = A \cdot X \quad (2)$$

where  $A$  is the conversion matrix that converts the input  $X$  (strain at LBSG) into the output  $F$  (global load or local stress). By the approximation, the input vector  $X$  ( $n \times 1$ ) is represented as the product of a modal response matrix  $M$  ( $n \times m$ ) consisting of responses by individual modes and a modal weight vector  $\xi$  ( $m \times 1$ ), where  $n$  is the number of LBSGs and  $m$  is the number of modes adopted.

$$X = M \cdot \xi \quad (3)$$

Similarly, the output vector  $F$  ( $l \times 1$ ) is also represented as the product of the modal response matrix  $B$  ( $l \times m$ ) and a modal weight vector  $\xi$  ( $m \times 1$ ), where  $l$  is the number of data to be estimated.

$$F = B \cdot \xi \quad (4)$$

The modal weights can be obtained by taking the inverse of the matrix  $M$  in Eq. (3).

$$\xi = M^{-1} \cdot X \quad (5)$$

In general, however, more LBSGs are used than the number of adopted modes. In these cases, the Moore-Penrose pseudoinverse matrix, which minimizes the quadratic errors, can be used.

$$\xi = (M^T \cdot M)^{-1} \cdot M^T \cdot X \quad (6)$$

By substituting Eq. (6) into (4), the relation between input  $X$  and output  $F$  is obtained.

### **DBM Induced by Regular Waves**

The key feature of the method is which type of DBM to be chosen. By F. Bigot, many types of distortion bases such as wet/dry vibration modes, unitary loads, regular wave, have been tested, and it is resulted that DBM by regular waves provide the best results [8-9]. That is, the distortion of overall structure can well be approximated by a linear combination of the quasi-static deformations of the ship caused by several regular waves. Each wave is represented by the combination of 3 parameters: headings  $\beta$ , frequencies  $\omega$  and phases  $\varphi$ . Bigot established the following wave selection process:

1. Set number of modes to be adopted,  $m$ .
2. Set the limit factor for the vector norm,  $\alpha$ .
3. Choose empirically a wave for the 1<sup>st</sup> mode (1<sup>st</sup> wave). The wave leading to maximum bending moment at midship is presented as an example. Calculate  $|\vec{X}_1|$  (norm of vector  $\vec{X}_1$ ), where  $\vec{X}_1$  consists of the input values, i.e., strain at LBSG induced by the 1<sup>st</sup> wave.
4. For the rest of waves, calculate  $|\vec{X}|$ . Leave only waves who satisfy  $|\vec{X}| > \alpha |\vec{X}_1|$ .

5. For the survived waves from 5, find the wave who minimizes the dot product between unit vectors,  $r_1 = (\vec{X}_1/|\vec{X}_1|) \cdot (\vec{X}/|\vec{X}|)$ . Define it as the wave for the 2<sup>nd</sup> mode (2<sup>nd</sup> wave). This is to find the 2<sup>nd</sup> wave as the least correlated one with 1<sup>st</sup> wave, secure orthogonality between modes.
6. Similarly with 6, for the rest of waves, find the wave who minimizes  $\text{MAX}(r_1, r_2, \dots, r_{j-1})$ . Define it as the wave for the  $j^{\text{th}}$  mode ( $j^{\text{th}}$  wave).  $r_i = (\vec{X}_i/|\vec{X}_i|) \cdot (\vec{X}/|\vec{X}|)$  is the correlation with the pre-selected waves ( $i=1, 2, \dots, j-1$ ).
7. Repeat 6 until  $j$  reaches  $m$ .

The above process only considers the orthogonality between input vectors of each modes, not the output vectors. Therefore, in order to ensure high accuracy of estimation, numerous case studies are required for the following: Configuration of LBSGs / Number of modes  $m$  (see 1) / Norm limitation factor  $\alpha$  (see 2) / Wave for the 1<sup>st</sup> mode (see 3).

### Index of Estimation Accuracy

The response due to a unit sinusoidal wave can be expressed in terms of amplitude and phase shift against the wave. During the case study, amplitude and phase shift of responses are estimated for each wave heading, frequency and location (for loads or stresses) by cases. There are differences between the pre-calculated responses and the estimated responses. To facilitate superiority comparison between cases, it is necessary to combine the differences from heading, frequency, and location into a single index. Previous studies have introduced root-mean-square-errors for long-term extreme value of stresses or damage via spectral-based fatigue analysis [8-12]. However, those values only uses the amplitude, so the estimation accuracy for phase shift could not be evaluated. There are also studies that have performed comparisons between time series [8-9, 13]. However, this is time-consuming, not suitable for studies with large number of cases.

In this study, the frequency domain error index (FDE) [14] is introduced. The amplitude and phase shift can be expressed in real and imaginary parts on the complex plane. For the pre-calculated and the estimated responses, differences of the real and imaginary part from wave headings, frequencies and locations are aggregated as following:

$$FDE = \frac{\sum_i \sum_j \sum_k \left( \sqrt{R_{c_{ijk}}^2 - R_{e_{ijk}}^2} + \sqrt{I_{c_{ijk}}^2 - I_{e_{ijk}}^2} \right)}{\sum_i \sum_j \sum_k \left( \sqrt{R_{c_{ijk}}^2 + I_{c_{ijk}}^2} + \sqrt{R_{e_{ijk}}^2 + I_{e_{ijk}}^2} \right)} \quad (7)$$

where,  $R_c$ ,  $R_e$ ,  $I_c$ ,  $I_e$  are real and imaginary part of calculated and estimated responses, and  $i, j, k$  are indices for wave heading, frequency and location. FDE has a value between 0 and 1, and can take into account both amplitude and phase differences through simple calculations. In FDE, responses with larger amplitude contribute more significantly. By multiplying the responses to the unit wave and the expected wave amplitudes, it is possible to weight the estimation accuracy for responses by waves with high occurrence probability. This is defined as weighted FDE (WFDE), and can be expresses as following:

$$WFDE = \frac{\sum_i \sum_j \sum_k \left( \sqrt{R_{c_{ijk}}^2 - R_{E_{ijk}}^2} + \sqrt{I_{c_{ijk}}^2 - I_{E_{ijk}}^2} \right) \times A_{ij}}{\sum_i \sum_j \sum_k \left( \sqrt{R_{c_{ijk}}^2 + I_{c_{ijk}}^2} + \sqrt{R_{E_{ijk}}^2 + I_{E_{ijk}}^2} \right) \times A_{ij}} \quad (8)$$

where,  $A_{ij}$  is the expected wave amplitude for headings ( $i$ ) and frequencies ( $j$ ), and can be calculated as following:

$$A_{ij} = \sum_u \sum_v \sqrt{2 \cdot S_{iuv}(\omega_j) \cdot \Delta\omega \cdot p_{iuv}} \quad (8)$$

where,  $S(\omega)$  is the power spectrum and  $p$  is the occurrence probability of wave for headings ( $i$ ), frequencies ( $j$ ) and significant wave height ( $u$ ), spectral peak period ( $v$ ).

Since waves with high occurrence probability generally contribute significantly to fatigue, WFDE can be used as a powerful index especially for fatigue life monitoring through stress estimation. In this study, case study of stress estimation introducing WFDE is conducted for the semi-submersible structure.

## CASE STUDY ON OFFSHORE SEMI-SUBMERSIBLE

### Target Structure

The semi-submersible structure shown in Figure 2 is considered for the case study. The hull is a four-column with ring-pontoon type and the topside deck is a stiffened-plate type. The approximate dimension of overall length/breadth and hull depth is 70 m and 40 m respectively. The approximate in-site displacement is 20,000 ton.

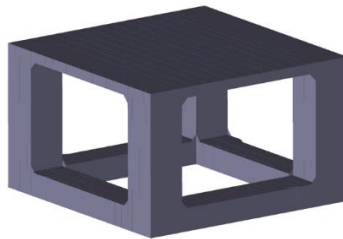


Figure 2. Semi-submersible structure for the case study (Components above topside deck is hidden)

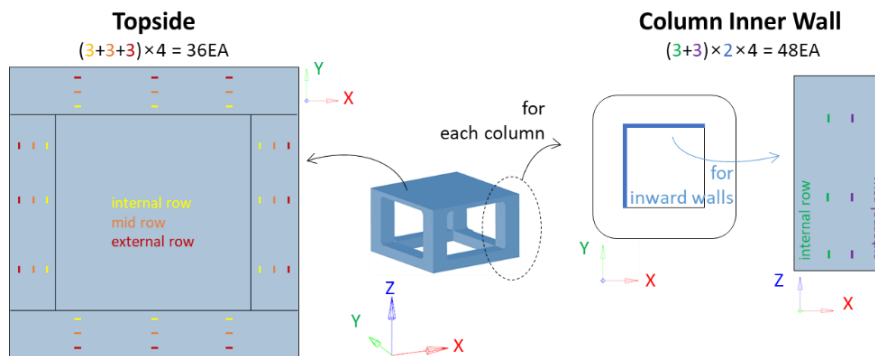


Figure 3. Candidate locations of LBSG placement

## Candidate Locations of LBSG Placement

To ensure rapid action in case of an abnormal sensor failure during operation, it is good to place LBSGs in easily accessible areas as possible. From this perspective, 36 locations on the topside and 48 locations on the inner wall of the column are selected as candidates for LBSG placement as shown in Figure 3.

## Locations for Local Stress Estimation

Based on the fatigue design result, total 20 locations with relatively low expected fatigue life are selected. Figure 4 shows 5 locations are selected at each column/pontoon.

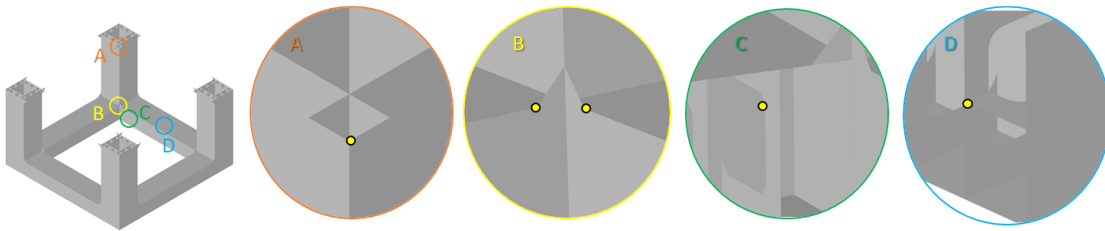


Figure 4. Candidate locations of LBSG placement

Locations are defined as nodal points in the structural finite element (FE) model. The stress component for each node is defined as the normal stress from direction (transformation with Mohr's circle), side (top/bottom) and element with minimum fatigue life expected by fatigue design.

## Case Study Set up

As mentioned above, cases for the study are defined with variation on the following features: Configuration of LBSGs / Number of modes  $m$  / Norm limitation factor  $\alpha$  / Wave for the 1<sup>st</sup> mode.

For the variation of LBSG configuration, candidate locations on the top side and column are extracted to make up the total number of LBSGs. Since adopting too many LBSGs would not be feasible, the cases are defined up to 52 LBSGs. The combinations on the number of LBSG are listed in TABLE I. The configuration is set to be symmetrical every 90 degrees. Therefore, the number of LBSGs for each case is a multiple of 4. The total number of variations on LBSG configuration is 650.

TABLE I. Combinations on the number of LBSG for the case study

<b>Topside</b>	<b>Numbers</b>	36	32	24	20	16	12	8	0				
	<b>Combinations</b>	1	3	4	3	6	5	5	1				
<b>Column</b>	<b>Numbers</b>	48	40	32	24	16	12	8	0				
	<b>Combinations</b>	1	6	1	6	6	4	4	1				
<b>Total</b>	<b>Numbers</b>	52	48	44	40	36	32	28	24	20	16	12	8
	<b>Combinations</b>	39	83	39	83	65	98	66	84	43	32	9	9

The variation of the number of modes  $m$  depends on the number of LBSGs, it is set to a multiple of 5, including  $n$  and  $n-1$ , where  $n$  is the number of LBSGs. For example, for the 24 LBSGs, the number of modes  $m$  shall be 24, 23, 20, 15, 10 and 5. Norm limitation factor  $\alpha$  is set to 0, 0.1, 0.2, ..., 0.9, so that the total number of variations is 10. For the wave for 1<sup>st</sup> mode, three waves are considered, each of which maximizes the vertical bending moment, transverse shear force and torsional moment at the center. In summary, a total of 155,820 cases were defined for the study.

## Results

TABLE II shows the cases resulting best estimation accuracy with respect to total number of LBSGs, Figure 5 shows WFDE for best cases with respect to 1) LBSG configuration (650EA) and 2) LBSG numbers on the topside and columns (46EA).

TABLE II. Cases with best estimation accuracy with respect to total number of LBSGs

LBSG Numbers			WFDE	Load by 1 <sup>st</sup> Wave	Mode # ( $m$ )	Norm Limit ( $\alpha$ )
Total	Topside	Column				
52	36	16	0.037	Torsion	51	0.3
48	36	12	0.044	Torsion	45	0.7
44	36	8	0.041	Bending	43	0.9
40	32	8	0.049	Bending	40	0.9
36	36	0	0.093	Shear	30	0.2
32	32	0	0.069	Bending	31	0.9
28	20	8	0.116	Torsion	28	0.4
24	24	0	0.134	Shear	23	0.8
20	20	0	0.164	Bending	20	0.7
16	16	0	0.211	Torsion	15	0.5
12	12	0	0.256	Torsion	12	0.8
8	8	0	0.340	Bending	7	0.9

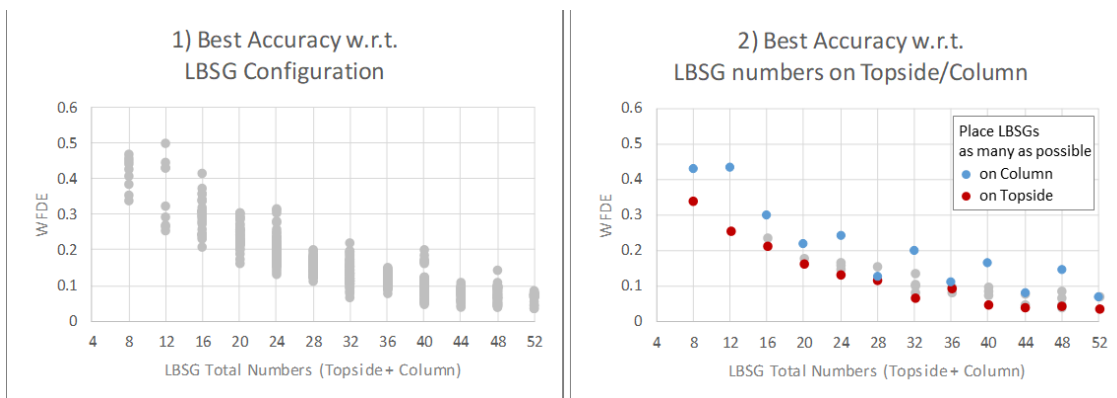


Figure 5. WFDE for cases with best estimation accuracy with respect to LBSG configuration (650 cases) and LBSG numbers on Topside/Column (46 cases)

The estimation accuracy generally improves (WFDE decreases) as the total number of LBSGs increases, however this does not always hold in the range where WFDE gets

smaller than some extent (#LBSG 32→36, 44→48). Trends of the 1<sup>st</sup> wave and norm limit ( $\alpha$ ) on the best estimation accuracy are not clearly found. From these results, the need for more case studies can be found, and accordingly the necessity of the efficient comparative index can be emphasized.

In most cases, the best estimation accuracy occurs when the (close to) maximum number of modes is used ( $m = n$  or  $m = n-1$ ). And the best estimation accuracy occurs when LBSGs are placed on the topside as many as possible, even if the total number of LBSGs is identical. Conversely, placing as many LBSGs as possible on columns resulted in the worst estimation accuracy. By this tendency, the variation on the number of modes and LBSG configurations can be reduced the variation on the other factors can be increased in the future studies.

## CONCLUSIONS

In this study, WFDE is introduced as the new index for efficiently conducting numerous case studies that are essential for wave-load induced DBM superposition method. With WFDE, both the amplitude and the phase shift of the responses can be considered through simple calculation. Using the index, a case study on semi-submersible structure is conducted and some trends of estimation accuracy with respect to the number of sensors is observed. It is expected that these trends will enable future studies to focus on more meaningful factors and be conducted more efficiently.

## REFERENCES

1. Korean Register (KR). 2023. "Guidance for Smart Systems"
2. American Bureau of Shipping (ABS). 2022. "Guide for Smart Functions for Marine Vessels and Offshore Unit"
3. International Maritime Organization (IMO). 1994. "Recommendations for the fitting of hull stress monitoring systems (IMO MSC/Circ.646)"
4. Korean Register (KR). 2023. "Rules for the Classification of Steel Ships", Pt. 9 Ch.6
5. American Bureau of Shipping (ABS). 2020. "Guide for Hull Condition Monitoring Systems"
6. Det Norske Veritas (DNV), "Rules for Classification", Pt.6 Ch.9
7. American Bureau of Shipping (ABS). 2020. "Advisory on Structural Health Monitoring: The Application of Sensor-based Approaches"
8. F. Bigot, Q. Derbanne, E. Baudin. 2013. "A review of strains to internal loads conversion methods in full scale measurements" presented at PRADS2013
9. F. Bigot, F.-X. Sireta, E. Baudin, Q. Derbanne, E. Tiphine, S. Malenica. 2015. "A novel solution to compute stress time series in nonlinear hydro-structure simulations", presented at OMAE2015
10. A. Mondoro, M. Soliman, D.M. Frangopol. 2016. "Prediction of structural response of naval vessels based on available structural health monitoring data", J. Ocean Engineering, 125: 295-307.
11. I. Drummen., L.Rogers, A.Benhamou, R.Hageman, K.Stambaugh. 2019. "Hull Structure Monitoring of a New Class of US Coast Guard Cutters", presented at the ASNE Technology, Systems & Ships.
12. Alexandru Andoniu, Jérôme de Lauzon, Remco Hageman, Pieter Aalberts, Didier L'Hostis, Alain Ledoux. 2021. "Validation of Spectral Fatigue Assessment of a West-Africa FPSO Using Full-Scale Measurements" presented at OTC2021
13. Seungyoung Lee, Kyoungtae Kim, Keunbok Song, Yun-ho Shin, Yoojeong Noh. 2024. "Optimizing Sensor Arrangement for Strain Mode-based Hull Monitoring System Application for Naval Ships", J. Asia-pacific Journal of Convergent Research Interchange, 10(8): 91-101
14. Dragovich, J. 1996. "An Experimental Study of Torsional Response of Reinforced Concrete Structures to Earthquake Excitation". Ph.D. Thesis Submitted to the Graduate College of the University of Illinois, Urbana-Champaign.