Enhancing Bridge Structural Health Monitoring: Integrating Physics-Based Knowledge and Machine Learning for Temperature Compensation

SABOOR KARIMI and MAX GUNDEL*

ABSTRACT

This study explores the application of a grey-box modeling approach for bridge SHM, integrating physics-based principles with machine learning (ML) techniques to enhance predictive accuracy. Using data collected from a tied-arch bridge, including acceleration, and temperature measurements, the study first evaluates different ML models, such as Linear Regression and Gradient Boosting Regressor, to eliminate the temperature effect from the natural frequency of a bridge cable. While purely data-driven models yield reasonable predictions, their accuracy improves when enhanced with prior physics knowledge.

Building upon this, a grey-box model is applied to further refine the temperature-compensation process, assessing its effectiveness in the SHM domain. The findings demonstrate that a grey-box model offers a more robust and reliable solution by leveraging fundamental physical principles alongside data-driven learning. This approach proves particularly beneficial in real-world SHM scenarios, where environmental and operational variability complicate damage detection and monitoring.

INTRODUCTION

One of the key parameters in SHM is the natural frequency of bridge elements, which can be significantly affected by environmental conditions, particularly

*Chair of steel structures, Helmut Schmidt University, Holstenhofweg 85, 22043 Hamburg, Germany

temperature. Variations in temperature influence the material properties and tension in cables, thereby altering their dynamic characteristics and potentially masking early signs of structural damage.

Traditional approaches to frequency prediction either rely purely on physical models or data-driven machine learning techniques. While physics-based models offer interpretability and a clear grounding in mechanical theory, they often fall short when dealing with complex, real-world variability. Conversely, machine learning (ML) models can capture nonlinear relationships and adapt to new data but may lack transparency and generalizability when not informed by domain knowledge [1].

To bridge this gap, greybox modeling integrates the strengths of both approaches by embedding physical principles into ML frameworks. This study demonstrates the application of greybox modeling to compensate for the temperature effects on the natural frequency of a cable in a tied-arch steel bridge. Using a combination of measured acceleration and temperature data, the study evaluates how prior knowledge, namely the calculated frequency and cable tension, can improve frequency predictions when incorporated into ML models. By this case study, the work underscores the potential of greybox modeling as a scalable tool for enhancing bridge monitoring systems.

PHYSICS-BASED MODEL

Cable structures, used in tied-arch steel bridges, behave as tensioned vibrating systems whose dynamic properties are influenced by physical parameters namely tension, cable length, and mass per unit length. A physics-based model provides an understanding of the system dynamics by capturing the relationship between temperature and natural frequency via known physical laws.

Fundamental Frequency Equation

The natural frequency of a taut, vibrating cable is primarily governed by its tension and linear mass density [2]. The equation for the k-th mode of vibration is:

$$f_k = \frac{k}{2L}\sqrt{\frac{N}{m_L}} \cdot \left[1 + \frac{2}{\xi} + \frac{1}{\xi^2}\left(4 + \frac{k^2\pi^2}{2}\right)\right]; \tag{1}$$

$$\xi = L\sqrt{\frac{N}{EI}}$$

Where ξ is the nondimensional stiffness and the EI is the bending stiffness. f_k represents the natural frequency for the kth vibration mode. The variable L and N denotes the length of the cable and the axial force due to tension in the cable, respectively. m_L refers to the mass per unit length of the cable. This relationship assumes idealized boundary conditions with constant tension and mass distribution.

Thermo-Mechanical Effects on Tension

In practice, the tension in a cable is not constant but varies with thermal expansion or contraction caused by temperature changes. The governing equation for temperature-dependent tension is:

$$N = EA(\varepsilon - \alpha_T \cdot \Delta T) \quad (2)$$

Here, the ε is the initial tension in the cable under reference temperature condition. The term E, A, and α_T are Young's modulus, cross-sectional area of cable, and thermal coefficient, respectively. The temperature difference from the reference condition is given by ΔT. Table I shows the characteristics of the selected cable bridge.

Practical Considerations and Assumptions

In practice, several factors complicate using physics-based models for frequency prediction. Real boundary conditions often differ from ideal assumptions, like pin-ended supports, affecting system dynamics. Additionally, unmodeled effects like damping, wind, and structural interactions introduce complexities not captured by basic models, leading to prediction errors.

Despite these limitations, the physics-based model serves as a critical component in the grey-box approach.

DATA PREPROCESSING AND FEATURE ENGINEERING

Data Analysis and Correlation

As part of the bridge monitoring system, a high-frequency accelerometer is attached at the cable midspan to capture vertical dynamics, and a temperature sensor is placed one-third up from the bottom anchorage. The accelerometer sampled at 1000 Hz, while the temperature sensor recorded slower-changing thermal conditions at 1 Hz. Data from both sensors are collected continuously over an extended monitoring period. While the accelerometer streamed data at a high sampling rate, raw acceleration signals are processed in 10-minute segments. Each 10-minute window is subjected to a Fast Fourier Transform (FFT) to extract the dominant vibration frequency, which is assumed to correspond to the cable's fundamental natural frequency.

To synchronize the two data sources, temperature readings within each 10-minute window were averaged to yield a single representative temperature corresponding to each frequency estimate. The final dataset comprised time-stamped records of natural frequency and corresponding cable temperature at 10-minute intervals, forming a time series suitable for exploration analysis and machine learning model development.

The initial visual inspection of the time series data does not reveal an apparent correlation between the two variables. Although both variables exhibited diurnal trends,

TABLE I. PARAMETERS OF THE SELECTED CABLE BRIDGE

Parameters	Value (units)
Cable length	6.711 (m)
Young's modulus	2.0×10^{11} (Pa)
Cross-sectional area	0.004536 (m^2)
Thermal expansion coefficient	1.2×10^{11} (/°C)
Mass per unit length	35.6 (kg/m)
Mode number	1 (-)

temperatures rising during the day and falling at night, with natural frequency fluctuating as well, their patterns are not closely synchronized in time. This lack of direct temporal alignment suggested the presence of a time-lagged effect.

To further investigate this possibility, a lagged Pearson correlation analysis is performed. The temperature time series is systematically shifted relative to the natural frequency series, and the Pearson correlation coefficient is computed at each lag. This analysis revealed a clear negative correlation peaking at a lag of 39, indicating that temperature changes affect the natural frequency approximately 390 minutes later interval.

This lag indicates that while cable temperature changes occur relatively quickly, the resulting mechanical response, in terms of axial force and vibration frequency, manifests with a noticeable delay. Importantly, this delay cannot be attributed to discrepancies between air temperature and cable temperature, as the temperature measurements are acquired directly on the cable. Instead, the lag is understood to arise from the complex thermo-mechanical interactions between the cable and the larger structural system of the bridge. Although the cable itself may heat or cool rapidly, the tension it experiences is also influenced by the thermal behavior of its boundary conditions, namely, the deck and arch structures to which it is anchored. These components, having significantly greater thermal mass and potentially different thermal conductivity and exposure conditions, exhibit slower temperature changes. This behavior has been reported in similar studies involving cable-supported structures [3]. Figure 2 shows measured frequency, physic-based frequency, and lagged temperature over time. To quantify this relationship, a regression analysis is done between temperature and natural frequency values across the entire dataset. The result indicated a negative correlation (see Figure 3).

Feature Engineering

To enhance the predictive capabilities of the grey-box model, a feature engineering strategy is adopted. The strategy is designed to integrate both domain-specific knowledge and statistical patterns inherent in the data. Three categories of features are engineered: basic features, time-based features, and rolling statistical measures.

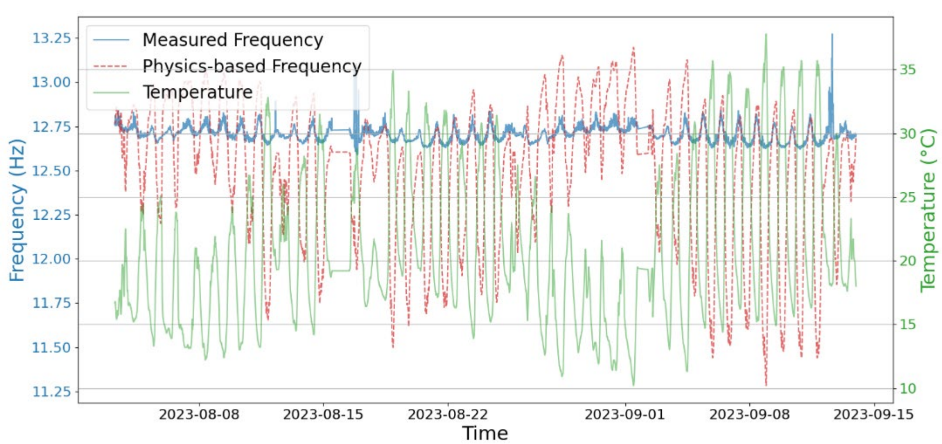

Figure 2. Measured frequency, physic-based frequency, and temperature over time

The basic features include cable temperature, which accounts for the thermal inertia effects observed in the cable. Additionally, features derived from tension, such as its square and cube, are included to model potential non-linear relationships between tension and frequency. Another input is the physics-based frequency calculated using the derived mechanical model. This value, based on known physical relationships, anchors the learning process within a physically meaningful framework. Interaction terms, such as the product of temperature with tension or physics-based frequency, are also introduced to capture the coupled effects and more complex interactions that occur in real-world bridge behavior.

Time-based features are incorporated to model the inherent cyclic and periodic behavior of environmental and operational conditions. Variables such as hour of the day, day of the week, and month are encoded using sine and cosine transformations to preserve their cyclical nature. This cyclic encoding allows the model to capture periodic fluctuations that may influence the structural response. Furthermore, interaction terms combining these time features with physical parameters, such as tension or temperature, help identify patterns that evolve with time.

Finally, rolling statistics [4] over a 24-hour window are computed for temperature, tension, and natural frequency. These include rolling means and standard deviations, which capture short-term trends and variations. This helps the model adapt to transient anomalies or gradual changes in system behavior, offering a more robust representation of the bridge's dynamic state.

Altogether, the feature set (see Table II) blends raw data, physics-derived insights, and time-aware patterns, providing the grey-box model with a rich, contextually informed foundation for accurate and interpretable prediction.

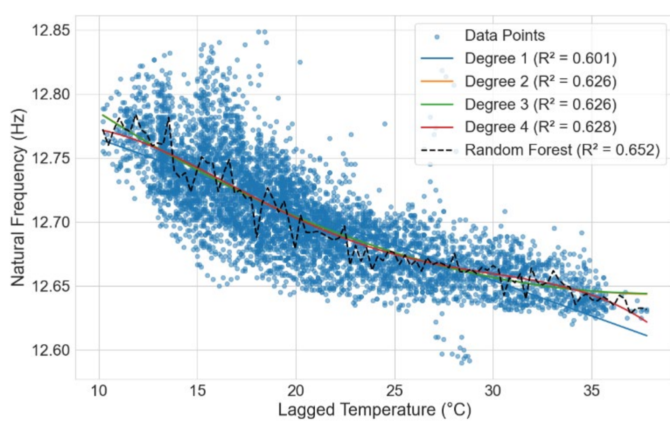

Figure 3. Comparison the regression models calculated between temperature and natural frequency.

TABLE II. LIST OF FEATURES

Features
• Temperature • Tension, Tension^2, Tension^3 • Physics-based frequency and its square • Interactions: temperature × tension, tension × physics frequency, and three-way interactions • Time features encoded cyclically: sin/cos transformations of hour, day, and month • Rolling mean and standard deviation over a 24-hour window for temperature and tension

MACHINE LEARNING MODELS AND GREYBOX IMPLEMENTATION

To evaluate the temperature-induced variation in the natural frequency of the bridge cable, two ML models are employed: Linear Regression (LR) and Histogram Gradient Boosting (HGB) [5]. The models are chosen for their interpretability and effectiveness in handling non-linear relationships, respectively.

Linear Regression provides a baseline for performance, offering a transparent and interpretable model where the contribution of each feature to the prediction can be understood. It is specifically useful when the relationship between inputs and the target variable is approximately linear. Given the physics-based formulation of the problem and the engineered features, LR helps identify the extent to which the physical and interaction features linearly relate to the observed frequency.

HGB, on the other hand, is a more sophisticated ensemble method capable of capturing complex, non-linear interactions among features. It builds multiple decision trees in a sequential manner, each tree aiming to reduce the error made by its predecessor [8]. This makes HGB suitable for modeling intricate dependencies, such as those between temperature, tension, time features, and natural frequency, particularly when these relationships deviate from ideal linearity.

Model training is conducted using a dataset split into training (80 %) and validation sets (20%), ensuring that model evaluation reflects generalization performance. Standard performance metrics such as Mean Square Error (MSE) and R-squared ($\mathscr{L}$^2) are used to assess model accuracy.

Model implementation

The grey-box approach integrates physics-based modeling with machine learning in a two-stage process. For comparative purposes, the performance of data-driven models is also briefly described. The data-driven models serve both as standalone (pure ML) predictors and as part of the grey-box residual modeling.

LR: A simple interpretable model with no need for hyperparameter tuning. It fits a linear function of the form:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \ldots + \beta_n x_n + \epsilon \tag{3}$$

where y is the predicted frequency, x_i are features, and β are the coefficients.

HGB: A powerful ensemble method using decision trees optimized via gradient descent [5]. It approximates:

$$F(x) = F_0 + \eta \Sigma_{t=1}^{T} h_t(x) \tag{3}$$

where η is the learning rate, and h_t are the weak learners (trees).

Grey-Box Modeling Approach

The grey-box approach consists of two main stages (see Figure 4):

Stage 1: *Physics-Based Frequency Estimation* Using the equation derived from tension theory (see Equation. 1). This produces a physically grounded estimate of the frequency.

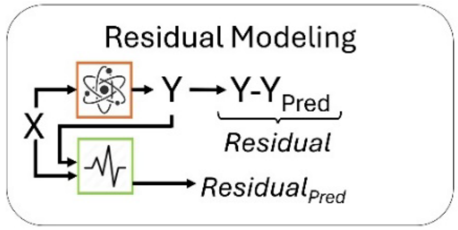

Figure 4. Schematic of the greybox model [1, 6].

Stage 2: *Residual Learning* Using ML The difference between the measured frequency and the physics-based frequency defines the residual. This residual is then modeled using LR and HGB trained on a comprehensive feature set derived through domain-specific feature engineering.

Training involves first calculating the physics-based frequency using domain equations, followed by computing the residuals as the difference between measured and physics-based frequencies. Features and target variables are then scaled using standardization. LR and HGB models are trained on these residuals. The predicted residuals are added back to the physics-based frequency to obtain final predictions, which are then evaluated on the test dataset.

RESULTS AND DISCUSSION

This chapter presents an evaluation of model performance, emphasizing the comparison between data-driven and greybox approaches using LR and HGB models.

The performance evaluation based on MSE and R^2 provides insights into the comparative strengths of the models.

The results indicate that the greybox approach yields lower MSE values across both LR and HGB models. Specifically, the MSE for data-driven LR stands at 0.000974, whereas the greybox LR reduces this to 0.000615, representing a 36.9% improvement. For the HGB model, the data-driven variant achieves an MSE of 0.000532, while the greybox version substantially lowers it to 0.000277, equating to a 47.9% improvement. Notably, the greybox-HGB combination emerges as the most accurate, with the lowest MSE recorded.

Across all model configurations, the greybox approach consistently enhances R^2 values. The data-driven LR achieves an R^2 of 0.557, improved to 0.720 with the greybox approach, indicating a 29.3% gain. Similarly, the R^2 of the data-driven HGB is 0.758, which increases to 0.874 for the greybox HGB, marking a 15.3% enhancement. These values underscore the greybox-HGB combination's superior capability in capturing variance within the dataset (see Figure 5).

Data-driven models show reasonable correlation with observed frequencies but tend to overestimate at higher values and display more scatter. Greybox models, particularly HGB, demonstrate tighter clustering around the ideal prediction line, better handling of extreme values, and more consistent performance. HGB handles non-linear relationships better, while LR provides more conservative estimates (see Figure 6).

The current models assume fixed physical parameters. Future improvements include dynamic parameter tuning, enhanced feature engineering, and incorporating additional physical effects.

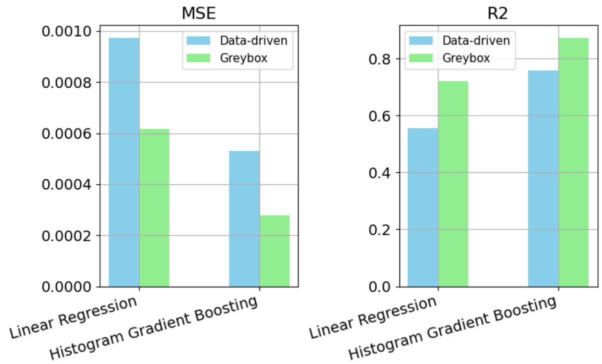

Figure 5. Compare MSE and R2 across models and approaches.

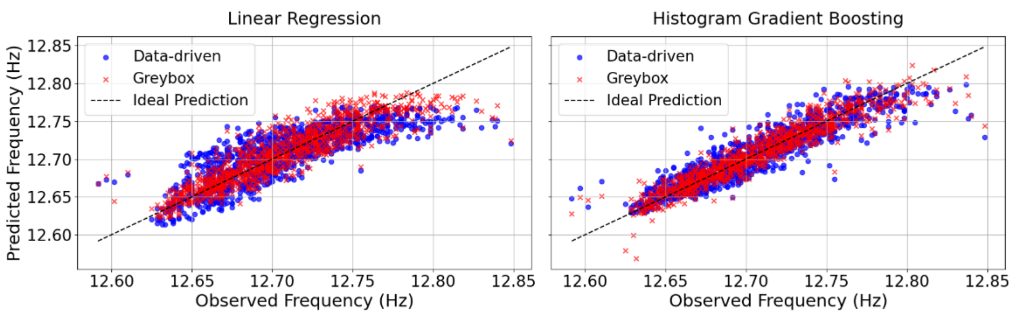

Figure 6. Observed vs Predicted frequencies for both approaches.

CONCLUSION

This study confirms the value of greybox modeling for bridge SHM by effectively combining physics-based understanding with machine learning. Integrating parameters such as tension and Physics-based frequency into learning models improves temperature effect compensation in frequency data.

The greybox approach, especially with HGB, yields the highest accuracy and reliability, outperforming data-driven methods across all metrics. This method proves particularly useful in real-world SHM contexts with complex environmental and operational variations.

In sum, greybox models strike an effective balance between predictive power and interpretability, making them a promising tool for advancing SHM systems. Their ability to embed physical knowledge enhances robustness and supports informed engineering decisions, paving the way for more reliable infrastructure monitoring.

ACKNOWLEDGEMENT

This research project is funded by the dtec.bw – Digitalization and Technology Research Center of the Bundeswehr. dtec.bw is funded by the European Union – NextGenerationEU.

REFERENCES

1. Karimi, S. and M. Gündel. 2024. "Advancing Structural Health Monitoring in Civil Engineering with Greybox Modelling: A Review," *E-Journal of Nondestructive Testing* 29 (7). https://doi.org/10.58286/29668.
2. Mordini, A., K. Savov, and H. Wenzel. 2008. "Damage Detection on Stay Cables using an Open Source-based Framework for Finite Element Model Updating." *Structural Health Monitoring*,7(2):91-102. doi:10.1177/1475921708089550
3. Magalhães, F., Cunha, Á., and E.Caetano. 2012. "Vibration based structural health monitoring of an arch bridge: From automated OMA to damage detection." *Mechanical Systems and Signal Processing*, 28, 212–228. https://doi.org/10.1016/j.ymssp.2011.06.011
4. Zivot, E., Wang, J. 2003. *Rolling Analysis of Time Series.* In: Modeling Financial Time Series with S-Plus®. Springer, New York
5. Guryanov, A. 2019 "Histogram-based algorithm for building gradient boosting ensembles of piecewise linear decision trees." In *Analysis of Images, Social Networks and Texts: 8th International Conference, AIST 2019, Kazan, Russia, July 17–19, 2019, Revised Selected Papers 8*, pp. 39-50. Springer International Publishing.
6. Cunha, B. Z., C. Droz, A. Zine, S. Foulard, and M. Ichchou. 2023."A review of machine learning methods applied to structural dynamics and vibroacoustic." *Mechanical Systems and Signal Processing* 200: 110535.