On Wave Field Kinematics Estimation from Structural Vibration Measurements Via Physics Informed Latent Force Models

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ABSTRACT

This study presents the application of the recently introduced Physics Informed Latent Force Models (PILFM) for estimating structural responses and wave kinematics (i.e., water particle velocity and acceleration) in the vicinity of an offshore structure, where a limited number of vibration responses are measured. This can enhance the accuracy of estimated structural responses, such as local stresses, that can be used for assessing the remaining fatigue life of the structure. PILFM integrates a model of the structural system that describes the structural vibration of the structure under wave loading, with known wave theories that describe the spatio-temporal behavior of the waves. A data-driven state space model is obtained based on the wave theories, and a Kalman Filter and Smoother are employed for the efficient estimation of the structural response and the wave kinematics. The current study presents a proof-of-concept of the proposed methodology, illustrated via a 2D structure subjected to stochastic waves. A few structural response measurements enable the estimation of the field kinematics in the vicinity of the structure and the full structural response.

INTRODUCTION

In the context of Structural Health Monitoring, a significant research effort has been dedicated to the estimation of the dynamic response during the operational life of structures of interest. This can serve different purposes, such as the assessment of the remaining fatigue life of the structure. In the case of offshore structures, the external wave and wind loading is unknown, and only a limited number of measured vibration responses can be used to estimate the full structural response. Different estimation methodologies have been developed for this purpose, mainly based on Bayesian filtering and smoothing techniques and modal expansion. A significant challenge that

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many techniques face, however, arises from the complex spatio-temporal behavior of the wave and wind loads, which results in a large number of unknown variables and makes the estimation problem ill-posed. The Physics Informed Latent Force Models (PILFM) framework, introduced in [1], addresses these challenges by incorporating established wave theories into the state estimation process. Unlike purely data-driven approaches, PILFM integrates prior knowledge of the physical phenomena generating the unknown loads within state-space modeling, enabling more accurate reconstruction of wave loading patterns while maintaining robustness against measurement limitations.

This study demonstrates the application of PILFM for estimating, together with the full structural response and unknown wave loads, the wave field kinematics near an offshore structure using limited vibration measurements.

PHYSICS INFORMED LATENT FORCE MODELS (PILFM)

Physics Informed Latent Force Models (PILFM) have recently been introduced in [1]. They include modeling of the unknown forces with a latent state space representation that reflects known theories describing the spatio-temporal behavior of the unknown loads. The name PILFM is a combination of the Latent Force Models [2], [3] and Physics Informed Machine Learning [4].

Consider the following linear time-invariant state space model describing the dynamic behavior of a structural system in discrete time:

$$\mathbf{z}_{k+1} = \mathbf{A}\mathbf{z}_k + \mathbf{B}\mathbf{p}_k + \mathbf{w}_k \tag{1}$$

$$\mathbf{d}_k = \mathbf{H}\mathbf{z}_k + \mathbf{J}\mathbf{p}_k + \mathbf{v}_k \tag{2}$$

where $\mathbf{z} \in \mathbb{R}^{n}$ is the state vector, $\mathbf{p} \in \mathbb{R}^{n_p}$ is the vector of unknown loads, $\mathbf{d} \in \mathbb{R}^{n_d}$ is the measurement vector, $\mathbf{A} \in \mathbb{R}^{n \times n}$ is the system matrix, $\mathbf{B} \in \mathbb{R}^{n \times n_p}$ is the input matrix, $\mathbf{H} \in \mathbb{R}^{n_d \times n}$ is the output matrix, $\mathbf{J} \in \mathbb{R}^{n_d \times n_p}$ is the direct feedthrough matrix, $\mathbf{w} \in \mathbb{R}^{n}$ is the process noise with covariance matrix $\mathbf{Q} \in \mathbb{R}^{n \times n}$, $\mathbf{v} \in \mathbb{R}^{n_d}$ is the measurement noise with covariance matrix $\mathbf{R} \in \mathbb{R}^{n_d \times n_d}$, and k indicates the time instant. The process noise and measurement noise are assumed to be zero-mean, Gaussian and white.

The load vector $\mathbf{p}$ is modeled as the output of a load system associated with the following noise-driven linear time-invariant state space model:

$$\mathbf{g}_{k+1} = \mathbf{A}^{\mathbf{L}}\mathbf{g}_k + \mathbf{w}^{\mathbf{L}}{}_k \tag{3}$$

$$\mathbf{p}_k = \mathbf{H}^{\mathbf{L}}\mathbf{g}_k + \mathbf{v}^{\mathbf{L}}{}_k \tag{4}$$

where $\mathbf{g} \in \mathbb{R}^{n_L}$ is the state vector of the load system, $\mathbf{A}^{\mathbf{L}} \in \mathbb{R}^{n_L \times n_L}$ and $\mathbf{H}^{\mathbf{L}} \in \mathbb{R}^{n_p \times n_L}$ are the corresponding system matrix and output matrix, respectively, $\mathbf{w}^{\mathbf{L}} \in \mathbb{R}^{n_L}$ is the driving process noise with covariance matrix $\mathbf{Q}^{\mathbf{L}} \in \mathbb{R}^{n_L \times n_L}$, while $\mathbf{v}^{\mathbf{L}} \in \mathbb{R}^{n_p}$ is the output noise with covariance matrix $\mathbf{R}^{\mathbf{L}} \in \mathbb{R}^{n_p \times n_p}$. The driving process noise and output noise are assumed to be zero-mean, Gaussian, white, and mutually uncorrelated.

The combination of Equations (1) and (2) with Equations (3) and (4) leads to the following augmented system:

$$\mathbf{z}^{\mathbf{a}}{}_{k+1} = \widehat{\mathbf{A}}^{\mathbf{a}}\mathbf{z}^{\mathbf{a}}{}_k + \widehat{\mathbf{w}}^{\mathbf{a}}_k \tag{5}$$

$$\mathbf{d}_k = \widehat{\mathbf{H}}^{\mathbf{a}}\mathbf{z}^{\mathbf{a}}{}_k + \widehat{\mathbf{v}}_k \tag{6}$$

with the following definitions:

$$\mathbf{z^a}_k = \begin{bmatrix} \mathbf{z}_k \\ \mathbf{g}_k \end{bmatrix}, \quad \widehat{\mathbf{A}}^{\mathbf{a}} = \begin{bmatrix} \mathbf{A} & \widehat{\mathbf{B}} \\ \mathbf{0} & \mathbf{A^L} \end{bmatrix}, \quad \widehat{\mathbf{w}}^{\mathbf{a}}_k = \begin{bmatrix} \widehat{\mathbf{w}}_k \\ \mathbf{w^L}_k \end{bmatrix}, \quad \widehat{\mathbf{H}}^{\mathbf{a}} = \begin{bmatrix} \mathbf{H} & \widehat{\mathbf{J}} \end{bmatrix} \tag{7}$$

$$\widehat{\mathbf{B}} = \mathbf{BH^L}, \quad \widehat{\mathbf{w}}_k = \mathbf{Bv^L}_k + \mathbf{w}_k, \quad \widehat{\mathbf{J}} = \mathbf{JH^L}, \quad \widehat{\mathbf{v}}_k = \mathbf{Jv^L}_k + \mathbf{v}_k \tag{8}$$

where the superscript "a" indicates the augmented system, while the symbol "$\frown$" indicates terms that depend on the load system. The augmented process noise, $\widehat{\mathbf{w}}^{\mathbf{a}}_k$, is associated with the covariance matrix $\widehat{\mathbf{Q}}^{\mathbf{a}} = \text{blkdiag}(\mathbf{Q} + \mathbf{BR^L B}^{\mathrm{T}}, \mathbf{Q^L})$ while the modified measurement noise $\widehat{\mathbf{v}}_k$ has covariance matrix $\widehat{\mathbf{R}} = \mathbf{R} + \mathbf{JR^L J}^{\mathrm{T}}$.

The state space model in Equations (5) and (6) can be used in combination with a Kalman Filter (KF) and Rauch-Tung-Striebel Smoother (RTSS) to perform the estimation. The equations of the KF and RTSS are not given here for brevity, but they can be found in [5].

The basic idea of PILFM is that the state space model in Equations (3) and (4) is crafted to reflect known theories that describe the loads' spatio-temporal behavior. However, for complex loads like wave loads, a direct state space representation is rarely achievable due to high complexity and nonlinearities. To address this challenge, a data-driven approach is adopted to obtain a stochastic state space model. This approach takes advantage of known physical theories by first employing them to generate training data (in this study, linear wave theory combined with irregular waves were used). System identification techniques are then applied to this data to derive an appropriate state space model. For this study, Dynamic Model Decomposition (DMD) was selected for its accuracy and efficiency. The detailed procedure for DMD-based state space identification can be found in [1] and is omitted here for brevity.

METHODOLOGY FOR ESTIMATING WAVE KINEMATICS

PILFM has been previously employed for the estimation of structural responses and wave loads [1]. The present study aims to expand the estimation by including wave field kinematics (i.e., water particle velocity and acceleration) in the vicinity of an offshore structure. To this end, we need to establish a relationship of the following form between the state vector of the wave load system and the desired quantities:

$$\mathbf{s}_k = \mathbf{H^s g}_k + \mathbf{v^s}_k \tag{9}$$

where $\mathbf{s} \in \mathbb{R}^{n_s}$ includes a generic quantity that needs to be estimated. In this work, we shall investigate the estimation of the horizontal water particle velocity acceleration. The covariance matrix error term $\mathbf{v^s}$ can be computed from the training data, similar to $\mathbf{R^L}$ [1].

To illustrate the idea, consider the desired quantity $\mathbf{s}$ to be the scalar field of horizontal water particle velocity evaluated in a spatial grid and gathered in the vector $\mathbf{u}$ (i.e., $\mathbf{s}$=$\mathbf{u}$). At a generic time step j, the load vector $\mathbf{p}_j$ can be stacked with $\mathbf{u}_j$ in a vector $\mathbf{x}_j$, which will constitute the building block of the training data:

$$\mathbf{x}_j = \begin{bmatrix} \mathbf{p}_j \\ \mathbf{u}_j \end{bmatrix} = \begin{bmatrix} \mathbf{p}_j \\ \mathbf{s}_j \end{bmatrix} \tag{10}$$

To favor an accurate identification of the dynamics, it is beneficial to augment the training data with its own derivatives or time-delayed versions of itself. In this study, time-delay embedding is performed by vertically stacking n_{lags} time-delays:

$$\mathbf{x}_j^{n_{lags}} = \begin{bmatrix} \mathbf{x}_j^{\mathrm{T}} & \mathbf{x}_{j-1}^{\mathrm{T}} & \dots & \mathbf{x}_{j-n_{lags}+1}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}. \tag{11}$$

The full training set $\mathbf{X}$ (which is a block Hankel matrix) can then be formed as follows:

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_{n_{lags}}^{n_{lags}} & \mathbf{x}_{n_{lags}+1}^{n_{lags}} & \mathbf{x}_{n_{lags}+2}^{n_{lags}} & \dots & \mathbf{x}_{n_{train}}^{n_{lags}} \end{bmatrix}, \tag{12}$$

and used in the DMD to identify the state space model. The following relationships can be used to connect the different quantities:

$$\mathbf{x}_j = \mathbf{\Phi}_{\mathbf{DMD}} \mathbf{g}_k + \mathbf{v}^{\mathbf{x}}{}_k \tag{13}$$

$$\mathbf{s}_j = [\mathbf{0}_{n_s \times n_p} \quad \mathbf{I}_{n_s \times n_s} \quad \mathbf{0}_{n_s \times (n_s+n_p)(n_{lags}-1)}] \mathbf{x}_j \tag{14}$$

where $\mathbf{\Phi}_{\mathbf{DMD}}$ contains the reduction basis (or mode shape vectors) computed with the DMD. The output matrix $\mathbf{H}^{\mathbf{s}}$ used in Equation (9) is defined as follows:

$$\mathbf{H}^{\mathbf{s}} = [\mathbf{0}_{n_s \times n_p} \quad \mathbf{I}_{n_s \times n_s} \quad \mathbf{0}_{n_s \times (n_s+n_p)(n_{lags}-1)}] \mathbf{\Phi}_{\mathbf{DMD}}. \tag{15}$$

NUMERICAL CASE STUDY: 2D MONOPILE OFFSHORE WIND TURBINE

To showcase the proposed methodology, a study was carried out consisting of a monopile wind turbine in parked condition (i.e., stationary blades) subjected to wave loads. This study aims to be a proof-of-concept of the proposed methodology.

Structure and wave loading

The offshore structure under consideration is shown in Figure 1. The wind turbine is the NREL 5MW reference wind turbine introduced in [6] while the monopile supporting structure has been replicated from the study in [7].

A 2D Finite Element (FE) model of the structure has been developed in OpenSees [8] using linear elastic beam-column elements. A total of 50 elements have been used for modeling the bottom part of the structure, which is subjected to waves. Such a discretization was used to accurately capture the spatial variability of the wave loads in the analysis and does not favor the estimation process per se. The nacelle and the blades have been modeled with a lumped translational and rotational mass. The distributed mass of the structure has been modeled using a consistent mass matrix formulation. Damping was modeled using the Rayleigh damping approximation, with a target

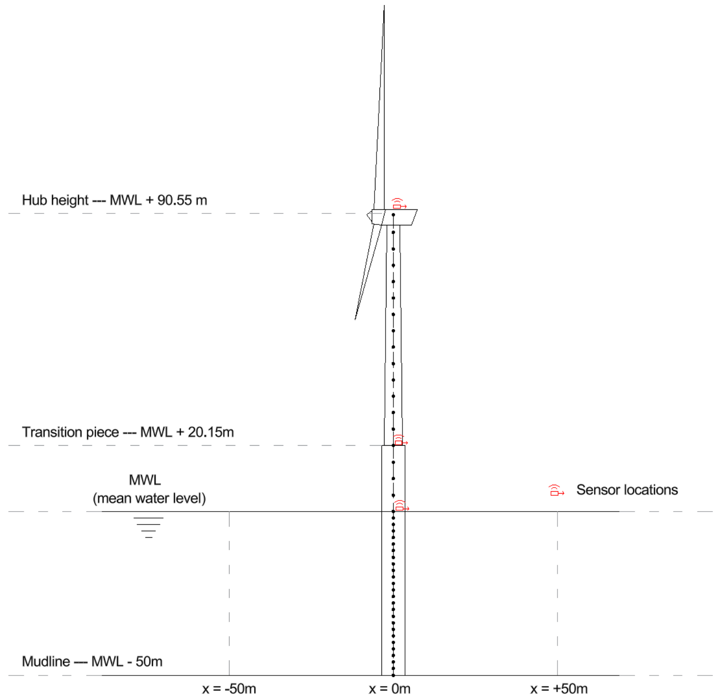

Figure 1: 2D monopile offshore wind turbine used in the current study. The nodes of the FE model representation are indicated as well as the location of the sensors of the structural response.

damping ratio of 2% for the first and third modes. The first 5 frequencies of the structure are 0.29Hz, 1.44 Hz, 3.86 Hz, 7.00 Hz, 7.56 Hz.

The wave field kinematics (i.e., water particle velocity and acceleration) were generated based on the linear (or Airy) wave theory. The JONSWAP spectrum with characteristic wave height of 9 m and peak period of 10 s was used to generate irregular (stochastic) waves. Once the wave kinematics were calculated, the Morison equation was used to compute forces and moments at all the submerged nodes of the FE model.

Available sensors and measurement noise

A sensor network including two sensor stations has been considered herein. Both stations are above the mean water level and include recordings of acceleration and velocity signals typically measured by accelerometers and geophones, respectively. The vibration data (signals) were calculated by performing time domain analysis of the FE model exposed to wave loads. Then, each signal was corrupted by adding Gaussian white noise with a standard deviation equal to 2% of the standard deviation of the corresponding clean signal. An investigation on the optimal placement of the sensors is considered outside the scope of the current study.

Kalman Filter tuning

The measurement noise is assumed to be given by the sensor manufacturer and hence known. The variances of the individual measurement signals are used in the matrix $\mathbf{R}$. Regarding the process noise covariance $\mathbf{Q}$, this was defined as an identity matrix multiplied by 1e-15. The role of $\mathbf{Q}$ is mainly to avoid numerical instability in calculations, and the authors did not notice a significant difference when using a different (small) value. The noise terms of the load system are computed from the training data. The reader is referred to [1] for further details on this aspect. The results shown in this study, corresponding to the smoothing mean estimates, were obtained using both the KF and the RTSS.

Training of PILFM

The training data and the hyper-parameters of the system identification and training procedure need to be defined. The following steps were followed:

- A set of time series data with a duration of 300 s and with the same wave characteristics as the "true" ones was generated including wave loads, horizontal water particle velocity, and horizontal particle acceleration. The particle velocity and acceleration were evaluated in a grid with 20 points along the vertical axis (i.e., z) and 41 along the horizontal axis (i.e., x), where the z was denser close to the surface to capture the variations with shorter length scale. The grid extends from the mudline to the surface in the z and from -50m to +50m (compared to the position of the wind turbine) in the x. The generated time series had a sampling period of 0.1 s. This is different from the sampling period of the dynamic analysis and consequent measurements, which is 0.01s.
- A total of 5 time-lags were used to build the block Hankel matrix $\mathbf{X}$ and a total of 40 modes (i.e., rank of the identified wave system) were selected.
- The resulting discrete dynamic model is resampled to a sampling period of 0.01 s. This involves the re-computation of the discrete-time poles of the dynamical systems contained in $\mathbf{A}^{\mathbf{L}}$ and the driving noise covariance matrix $\mathbf{Q}^{\mathbf{L}}$ (see [1]).

An investigation, not shown here for brevity, was conducted demonstrating that a larger number of time-lags and/or a larger number of modes did not provide significantly different results. Such an investigation concerns the activity of system identification and is considered to be outside the scope of this study. Note that the time series data used for the training is different from the one generating the true unknown kinematics and loads used for the analysis and the subsequent estimation.

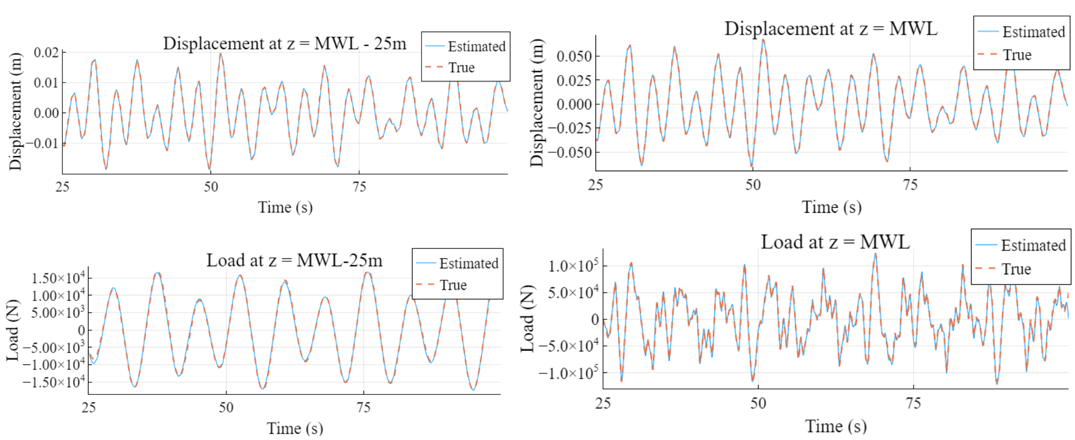

Figure 2: True and estimated displacements (top) and loads (bottom) at z=-25m (left) and z=0m.

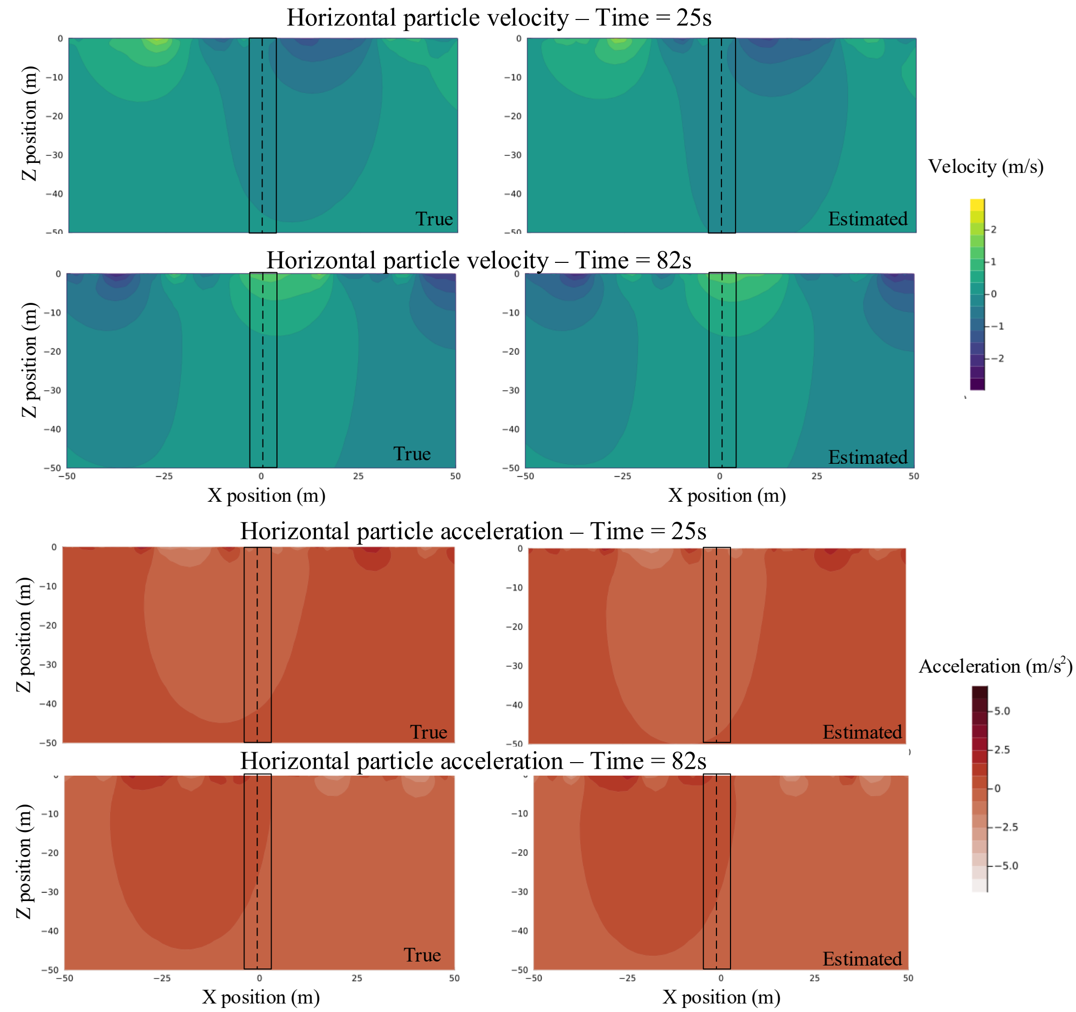

Figure 3: True(left) and estimated (right) wave field horizontal velocity (top) and acceleration (bottom) at time=25s and time=82s. The solid lines and the dashed line indicate the location of the wind turbine and its axis.

Results: response and load estimation

Figure 2 shows the estimation of the displacement at the top of the offshore wind turbine structure as well as at 25 and at 0 m below the mean water level. All the estimated displacements are very accurate since they are almost identical with the "true" displacements, calculated herein directly from the time domain analysis. Similar results are seen for the loads at the same heights.

Results: estimation of wave field kinematics

Figure 3 shows the true and estimated fields of horizontal water particle velocity (horizontal and vertical) and acceleration (horizontal and vertical) at time instants 25s and 82s. These plots are representative of the overall accuracy of the estimated wave kinematics, and they reveal that the estimation of the field kinematics is not perfect, but the main spatial and temporal patterns have been successfully captured.

CONCLUSIONS

This paper presented a proof-of-concept study on the estimation of structural response, unknown wave loads, and unknown wave kinematics using the recently introduced Physics Informed Latent Force Models (PILFM). The methodology is a direct extension of the original PILFM introduced in [1].

The case study consists of a 2D wind turbine model subjected to wave loads while two sensor stations, measuring structural vibration responses, were assumed as the available source of information. From the vibration measurements, the estimation of the full structural response, the unknown loads, and the wave field kinematics in the vicinity of the structure is performed. The estimation of the response and loads is very accurate, with the response being estimated nearly perfectly. The estimation of the wave field kinematics clearly captures the global spatio-temporal patterns, while the local variations are not perfectly estimated. This is expected, since the measured structural response is drastically lower dimensional than the full field kinematics that is being estimation, and the information to estimate the latter in all points in space is expected to be much more than that contained in the measured response. We see the results shown in this study as a promising first step. Further and more elaborate studies will follow to address existing challenges and consider more realistic scenarios with higher complexity.

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