

# Slow Dynamics for Concrete Strength Evaluation

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## ABSTRACT

This study explores the use of slow dynamics behavior to estimate the compressive strength of concrete. Slow dynamics, a nonlinear elastic phenomenon observed in heterogeneous materials like concrete, involves stiffness reduction under conditioning followed by a logarithmic recovery over time. Concrete specimens with compressive strengths of 32, 40, 49, and 56 MPa were tested. Each was subjected to a 6 kN load while ultrasonic signals were monitored to track relative velocity changes ( $dv/v$ ) during recovery.

Higher strength specimens showed larger velocity drops and faster recovery. The 56 MPa samples had  $dv/v$  drops that were 2.23, 1.80, and 1.37 times greater, and recovery rates that were 2.09, 1.84, and 1.37 times faster, than those of the 32, 40, and 49 MPa groups. In contrast, ASR damaged specimens exhibited greater sensitivity to conditioning but recovery rates that did not scale with the velocity drop, unlike the proportional behavior observed in intact samples.

## INTRODUCTION

Accurate in-situ assessment of concrete is critical in evaluation of structural health, with compressive strength in particular being a key predictor of service life. Reliable estimation of compressive strength is essential. Typical strength evaluation methods offer distinct advantages and drawbacks. Coring, for example, is a method that can provide accurate strength measurements, but the results are localized. Common nondestructive methods, such as ultrasonic pulse velocity and impact-echo, estimate strength by measuring linear acoustic properties, but often lack sufficient sensitivity. Nonlinear methods, which monitor the changes of those linear features under varying strain levels, are of interest due to their high sensitivity in evaluating materials with complex microstructures, such as concrete. One nonlinear feature observed in concrete is its rapid decrease and slow recovery of material properties following a perturbation, a behavior known as slow dynamics.

Slow dynamics is a nonlinear elastic behavior observed in various heterogeneous

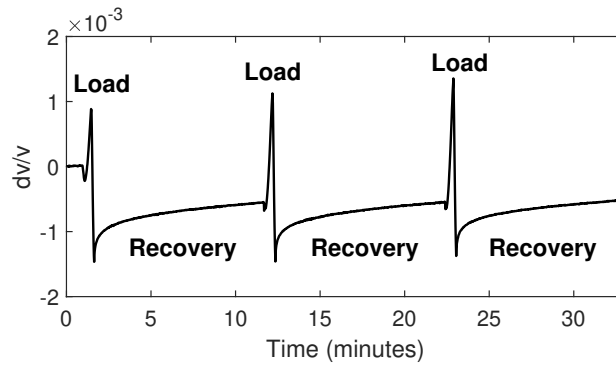


Figure 1. Conditioning and recovery of concrete specimen.

materials such as concrete, where stiffness decreases under conditioning (mechanical or thermal), followed by a  $\log(\text{time})$  recovery toward its initial state. First identified in resonance experiments on rocks, this behavior reflects material “memory” to loading. It is characterized by a conditioning phase (excitation resulting in reduction in stiffness) and a relaxation phase (stiffness recovery). Extensive research has been performed in understanding slow dynamics and its sensitive behavior to internal structural complexities. [1–9]

This behavior has been investigated in several materials through resonance tests [2, 10], nonlinear resonant ultrasound spectroscopy [11–15], dynamic acousto-elastic tests [16, 17], and through monitoring ultrasonic relative velocity change through coda wave interferometry [18], among other methods. For concrete in particular, it has been studied in relation to damage characterization in mechanical, ASR, and thermal damaged specimens [10, 18–21].

This study proposes monitoring the slow dynamics behavior of concrete for the evaluation of concrete strength. Concrete specimens of different strengths were cast and their recovery following the same conditioning was monitored by recording their relative velocity change ( $dv/v$ ). Fig. 1 shows the response of a concrete specimen to an applied compressive load, and its subsequent recovery, through monitoring its relative velocity change.

## MATERIALS AND METHODOLOGY

### Test Specimens

The four mix designs, found in Table 1, were used to cast the concrete prisms (75 mm  $\times$  75 mm  $\times$  285 mm) tested in this study. For each mix design, three concrete prisms were cast and used for monitoring slow dynamics behavior. Additionally, several cylinders were cast from each batch to verify the compressive strength of each mix (32.2 MPa, 40.4 MPa, 48.8 MPa, and 56.3 MPa).

### Stretching Technique for Signal Analysis

TABLE I. Mix designs for specimens tested in this study.

Ingredient	Mix 1	Mix 2	Mix 3	Mix 4
Cement (kg/m <sup>3</sup> )	263	341	421	534
Coarse Aggregate (kg/m <sup>3</sup> )	1100	1100	1100	1100
Fine Aggregate (kg/m <sup>3</sup> )	793	727	659	565
Water (kg/m <sup>3</sup> )	160	160	160	160
HRWR (mL)	-	-	-	50
w/c	0.61	0.47	0.38	0.30
28-day $f'_c$ (MPa)	32.2	40.4	48.8	56.3

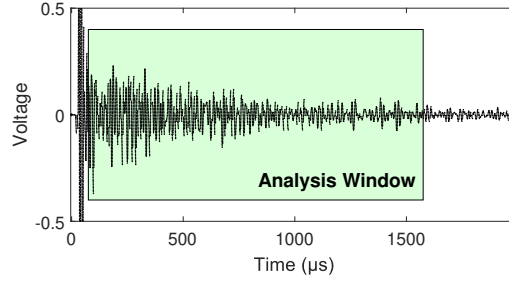


Figure 2. Typical time domain signal.

To evaluate ultrasonic signal changes in concrete, researchers commonly apply the stretching technique [22]. By calculating the cross-correlation coefficient (CC) between the reference signal  $s_0(t)$  and the perturbed signal  $s_1(t)$ , the relative velocity change can be found. The calculation is performed as:

$$CC(\varepsilon) = \frac{\int_{t_a}^{t_b} s_1[t(1+\varepsilon)] s_0(t) dt}{\sqrt{\int_{t_a}^{t_b} s_1^2[t(1+\varepsilon)] dt \int_{t_a}^{t_b} s_0^2(t) dt}} \quad (1)$$

where  $[t_a, t_b]$  defines the chosen time window of the signal for the analysis, and  $\varepsilon$  is the stretching factor. For the purpose of monitoring slow dynamics,  $s_0(t)$  refers to an ultrasonic signal recorded prior to conditioning, and  $s_1(t)$  refers to a signal during recovery. The relative velocity change can be determined from the relative time change by  $dV/V = -dt/t = \varepsilon_{max}$ , where  $\varepsilon_{max}$  is the stretching factor that maximizes the CC. A typical time domain signal used during analysis in this study is shown in Fig 2.

## Experimental Setup

Specimen conditioning was initiated by a compressive force supplied by a universal test machine (Instron 6800 series). Specimens were loaded to a peak load of 6 kN, with a 300 N/s loading rate during the loading phase, and a 600 N/s rate during the unloading phase. Prior to (at rest), during (conditioning), and after (recovery) loading, ultrasonic signals were acquired.

The experimental setup for the ultrasonic measurements is shown in Fig. 3. Two shear transducers (Panametrics V1548, 0.1 MHz) were installed on the concrete prisms

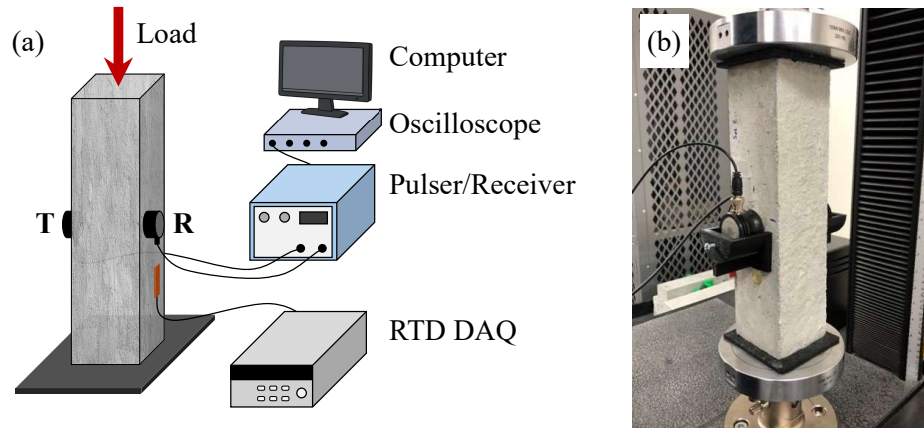


Figure 3. Experimental setup for monitoring slow dynamics behavior.

operated in the through-transmission mode. The transducers were driven by pulse (0.1 MHz center frequency) supplied by a pulser/receiver (Olympus 5077PR). The signals were digitized by an oscilloscope (PicoScope 5444A) with a sampling frequency of 250 MHz and averaged 8 times.

Thin film RTDs sensors (TE Connectivity,  $1\text{k}\Omega$ , NB-PTCO-126, 4-wire configuration) were used to monitor the temperature of the prism and the surrounding environment during testing. The temperature data was acquired using a multiplexer module (Agilent 349702A).

## RESULTS AND DISCUSSION

### Concrete Strength Evaluation

The relative velocity change during recovery across the four different strength groups is shown in Fig. 4. Three specimens from each group were tested, and the average value  $dv/v$  value of the group at each time step is shown by the bold line. The log regression coefficients of the average group responses are displayed. Stronger specimens have a larger initial relative velocity drop and experience a faster recovery rate.

Compared to specimens with compressive strengths of 32, 40, and 49 MPa, the strongest group ( $f'_c = 56$  MPa) showed an average relative velocity drop ( $dv/v$ ) that was 2.23, 1.80, and 1.37 times greater, respectively. Likewise, their recovery rate was 2.09, 1.84, and 1.37 times faster. The stronger specimens have a higher initial stiffness and accordingly experience a larger relative drop in rigidity from the conditioning than their weaker counterparts. Likewise, during recovery the stronger specimens stiffness rises back toward its original value at a faster rate, that is nearly proportional to the difference in  $dv/v$  drop.

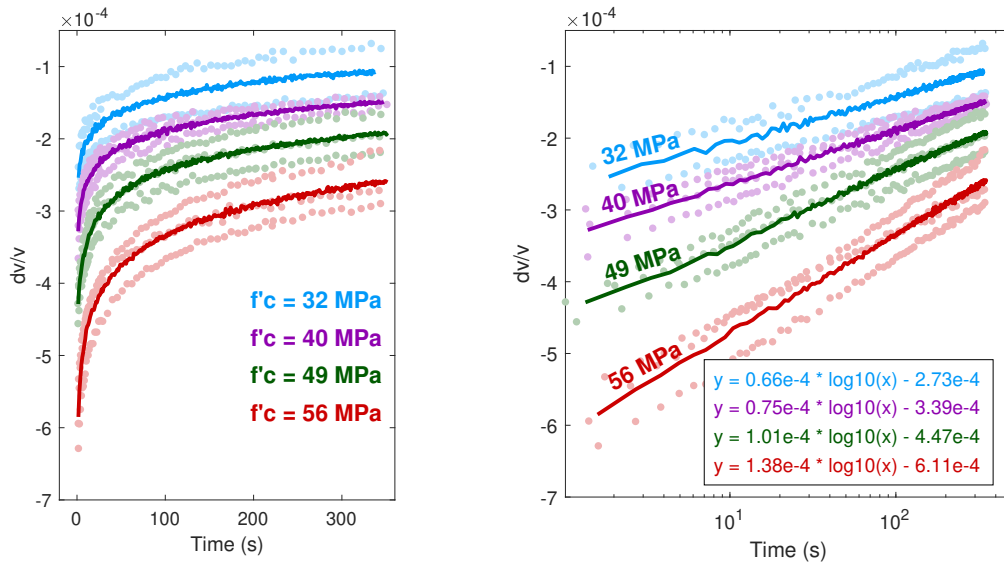


Figure 4. Relative velocity change history during recovery phase (right is log(x) scale).

As displayed from the behavior in Fig. 4, for the specimens in this study, higher concrete compressive strength results in larger relative velocity drops and recovery rates from the same perpetuance. However, the presence of damage in specimens is known to result in similar behavior (higher material nonlinearity and higher sensitivity to conditioning). A study by Larose et al. [18] found that mechanically damaged concrete specimens experienced an initial relative velocity drop nearly ten times greater than that of undamaged companion samples.

For the purpose of material evaluation, concrete strength and damage seem to be "competing" factors in terms of slow dynamics response. That is, when testing two undamaged samples, a larger relative velocity drop and faster recovery rate seems to be indicative of a stronger specimen. However, if testing the same specimen at different times, once before damage occurred and once after, the presence of damage would also manifest as a larger relative velocity drop and faster recovery rate.

To investigate this behavior on the specimens in this study, two specimens with moderate ASR damage were tested and their results compared with the specimens of the four groups of this study. The ASR damaged specimens are of the same dimension and similar mixes aggregate gradations to the four undamaged groups. Companion samples of the ASR damaged mixes were found to have compressive strengths of approximately 35 MPa. The results of tests performed on the ASR damaged are shown in Fig. 5. It can be seen that the damaged specimens have considerably higher material nonlinearity, and recover on a considerably different time scale than the intact specimens. While the competing effect of damage is seemingly a challenge for strength evaluation, it seems that the intact specimens operate in a narrow range of behavior when compared to the damaged specimens. Additionally, unlike the intact specimens, the damaged specimens do not recover faster at a rate proportional their relative velocity drop. Compared to the 56 MPa group, the relative velocity change of the damaged specimens dropped 1.96 and 2.58 times more, but the recovery rate was only 1.42 and 2.00 times higher.

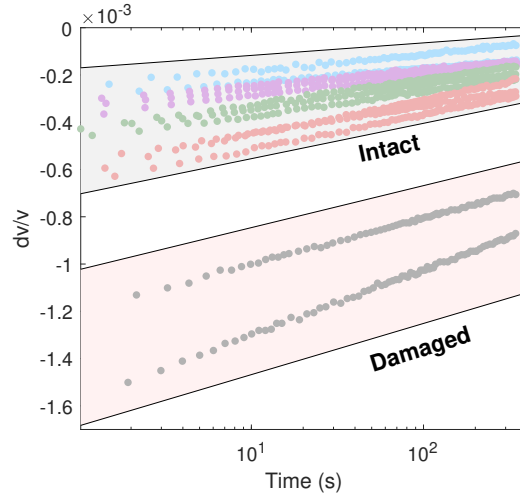


Figure 5. Relative velocity change history including ASR damaged specimens.

### Experimental Repeatability

For this behavior to be beneficial for strength evaluation, tests performed on the same specimens should yield similar results. Therefore, to evaluate the repeatability of the recovery behavior, four consecutive tests were repeated on the same specimen. After each test, the specimen recovered for five hours, allowing time for the specimen to return near its initial state. The results are shown in Fig. 6 and Table II.

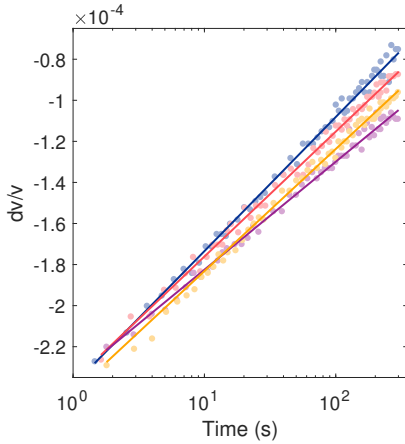


Figure 6. Relative velocity change of test repeated on same specimen.

$dv/v = a \times \log(t) + b$		
Test	a	b
1	$6.53 \times 10^{-5}$	$-2.39 \times 10^{-4}$
2	$5.26 \times 10^{-5}$	$-2.35 \times 10^{-4}$
3	$6.08 \times 10^{-5}$	$-2.37 \times 10^{-4}$
4	$5.95 \times 10^{-5}$	$-2.43 \times 10^{-4}$
<b>Mean</b>	$5.96 \times 10^{-5}$	$-2.39 \times 10^{-4}$
<b>SD</b>	$0.54 \times 10^{-5}$	$0.32 \times 10^{-4}$

TABLE II. Results for the repeated tests.

### CONCLUSION

In this work, the slow dynamics response of concrete specimens of varying strengths was investigated. To explore this behavior, ultrasonic signals were collected on several specimens from four groups (compressive strengths of 32 MPa, 40 MPa, 49 MPa, and

56 MPa) as they were loaded and during recovery. The method is efficient and can be applied in field settings, as it involves only a brief perturbation and observation of the material's natural recovery. Consistent behavior within strength groups indicate its potential usefulness for strength evaluation. The strongest specimens ( $f'_c = 56$  MPa) showed an average relative velocity change drop ( $dv/v$ ) that was 2.23, 1.80, and 1.37 times greater than those with compressive strengths of 32 MPa, 40 MPa, and 49 MPa, respectively. Likewise, the recovery rate of the strongest specimens was 2.09, 1.84, and 1.37 times faster than the weaker groups. Damaged materials showed increased sensitivity to conditioning, as shown by a larger  $dv/v$  drop in two ASR-damaged specimens. However, despite the significant drop in relative velocity change, the damaged specimens did not exhibit a similarly fast recovery. This contrasts with the behavior of intact specimens, where larger velocity drops in stronger samples were accompanied by proportionally faster recovery rates. Further analysis needs to be performed regarding how the presence of damage may mask recovery features related to material strength, as the nonlinearity they induce can be orders of magnitude greater. In future work, these findings will be validated on intact specimens of several different aggregate types and mix designs, investigating the influence of features other than compressive strength on the recovery behavior.

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