Value of Information Analysis for Structural Health Monitoring Considering Multiple Damage Modes

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ABSTRACT

The Value of Information (VoI) framework is a crucial component in evaluating, designing, and optimizing the Structural Health Monitoring (SHM) system by assessing its overall benefit in lifecycle asset management. It focuses on conducting a cost-benefit analysis of decision-making throughout the lifecycle. In practice, the overall damage in a structure consists of multiple damage modes that collectively determine its health. Therefore, it is essential to develop a VoI framework that can incorporate multiple damages and their evolution—first, to evaluate the performance of an SHM system, and second, to optimally design the SHM system. A multi-damage-mode VoI framework is proposed, and its application to the lifecycle assessment of a miter gate structure is discussed.

INTRODUCTION

Broadly, Structural Health Monitoring (SHM) has three main objectives: (1) *Diagnosing* the current condition of a structure using noisy sensor measurements; (2) Forecasting the future evolution of the structural state (*prognostics*); and (3) Supporting well-informed *decision-making* based on the outcomes of diagnostics and prognostics.

When implemented effectively, SHM enables more informed and potentially optimal decisions. However, such efforts are only justifiable if the benefits of SHM outweigh the associated costs. It is therefore essential to evaluate the economic merit of SHM by quantifying the *Value of Information (VoI)* derived from the acquired data—or features extracted from the data.

In our previous work [1, 2], we developed a consequence-based VoI framework for a *single damage mode* in a miter gate structure, specifically focusing on the loss of contact in quoin blocks (a boundary-related damage referred to as *gap length*). However, real-world structural deterioration rarely manifests as a single mode. A more realistic SHM framework must account for multiple co-occurring damage modes. In such cases, defining a collective structural state—one that meaningfully integrates multiple damage modes—becomes critical. This requires a paradigm shift in how structural state is represented and interpreted.

For example, with only one damage mode (e.g., loss of boundary contact), the structural state can be characterized by the posterior distribution of the gap length given the observations. However, if we consider multiple simulated damage modes—such as *gap length* and *uniform material loss due to corrosion*—the structural state must jointly reflect both. Moreover, since different damage modes can influence each other, a more general formulation is needed to define a collective structural state. One possible way to define the collective structural state is by using the cumulative probability of failure, considering limit states associated with multiple damage modes.

The second task is to define consequence functions for a range of possible decisions and use them to evaluate the VoI. In this work, we demonstrate how to compute the cumulative probability of failure of the structure considering multiple damage modes and adapt the consequence-based VoI framework developed in [1] to accommodate multi-damage scenarios. We also briefly present the generalized VoI formulation. Notably, the proposed framework naturally allows for incorporating the risk profile of the decision-maker directly into the consequence functions [2].

VALUE OF INFORMATION

In a general setting, the use of an SHM strategy is expected to yield net savings over the lifecycle of a structure, relative to a less informed decision-making policy, after accounting for all associated costs [1,3,4]. Ignoring the cost of the SHM system itself, let $C_{\text{saved}_{\text{LC}}}(\wp_e|\wp_{\sim e})$ denote the expected cost savings, defined as the difference in decision-related costs (i.e., the costs incurred from taking specific actions at specific times over life of the structure) between the non-SHM (or in general, a different mechanism than strategy e) decision policy $\wp_{\sim e}$ and SHM-informed strategy e with policy $\wp_e$ over the lifecycle (LC).

$$C_{\text{saved}_{\text{LC}}}(\wp_e|\wp_{\sim e}) = C_{\sim e_{\text{LC}}}(\wp_{\sim e}) - C_{e_{\text{LC}}}(\wp_e). \tag{1a}$$

Here, $C_{\sim e_{\text{LC}}}(\wp_{\sim e})$ denotes the expected lifecycle cost of decisions made under a strategy different from e, following decision policy $\wp_{\sim e}$; $C_{e_{\text{LC}}}(\wp_e)$ denotes the expected lifecycle cost under an SHM-informed strategy e with the corresponding decision policy $\wp_e$. The net savings—and therefore the VoI—depend not only on the SHM system e, but also on the associated decision policy $\wp_e$. Therefore, the SHM strategy is denoted by the pair $(e, \wp_e)$. The obvious requirement for any SHM strategy e to be considered better than the alternative strategy $\sim e$ to which it is compared is that it results in relative positive net savings, i.e., $C_{\text{saved}_{\text{LC}}}(\wp_e|\wp_{\sim e}) > 0$.

However, this condition alone is not sufficient. The net savings $C_{\text{saved}_{\text{LC}}}(\wp_e|\wp_{\sim e})$ must also exceed the expected investment of strategy e. The investment associated with an SHM related strategy e consist of installing, operating, and maintaining the SHM system e over its deployment period. This investment is denoted by $C_{\text{invested}_{\text{LC}}}(e)$. Note that $C_{\text{invested}_{\text{LC}}}(e)$ refers to the cost associated with the SHM system itself, and not the decision-related costs associated with the policy $\wp_e$, which is already accounted for in $C_{e_{\text{LC}}}(\wp_e)$. The net benefit of the information acquired through an SHM strategy $(e, \wp_e)$ can therefore be expressed using the following two Value of Information (VoI) metrics:

$$\text{EVoI}_{\text{LC}}(e, \wp_e|\wp_{\sim e}) = C_{\text{saved}_{\text{LC}}}(\wp_e|\wp_{\sim e}) - C_{\text{invested}_{\text{LC}}}(e), \tag{2a}$$

$$\lambda_{LC}(e, \wp_e|\wp_{\sim e}) = \frac{C_{\text{saved}_{\text{LC}}}(\wp_e|\wp_{\sim e})}{C_{\text{invested}_{\text{LC}}}(e)}. \tag{2b}$$

Here, $\text{EVoI}_{\text{LC}}(e, \wp_e|\wp_{\sim e})$ and $\lambda_{LC}(e, \wp_e|\wp_{\sim e})$ represent the *Expected Value of Information* and *risk-adjusted reward* (a metric proposed in [1]), respectively, of an SHM strategy $(e, \wp_e)$, relative to the baseline strategy (or policy) $\wp_{\sim e}$ obtained using pre-posterior decision analysis. For an SHM strategy $(e, \wp_e)$ to be considered viable, a strictly necessary condition is to have $\text{EVoI}_{\text{LC}}(e, \wp_e|\wp_{\sim e}) > 0$ or equivalently

$\lambda_{LC}(e, \wp_e|\wp_{\sim e}) > 1$. To estimate $C_{\text{saved}_{\text{LC}}}(\wp_e|\wp_{\sim e})$, the consequence functions will be developed in a later section using normalized costs. Therefore, $C_{\text{invested}_{\text{LC}}}(e)$ must also be normalized using the same factor. In that regard, using $\lambda_{LC}(e, \wp_e|\wp_{\sim e})$ is preferable, as it is unitless and does not represent an absolute dollar amount.

Equation (17) of Chadha et al. [5] provides an expression for $C_{\text{invested}_{\text{LC}}}(e)$. However, the formulation of $C_{\text{saved}_{\text{LC}}}(\wp_e|\wp_{\sim e})$ in Eq. (18) of Chadha et al.'s [5] represents a special case of lifecycle net savings, derived using Expected Utility Theory for decision-making under a single damage mode. In this paper, we focus on defining the collective structural state for multiple damage modes and on formulating the consequence cost function as a function of this collective state—an essential component in computing $C_{\text{saved}_{\text{LC}}}(\wp_e|\wp_{\sim e})$.

DEGRADATION MODEL FOR MULTIPLE DAMAGE MODES

For demonstration purposes, we numerically simulate two damage modes to mimic the damage evolution in the miter gate structure: (1) loss of boundary contact at the quoin block (referred to as gap length), and (2) loss of uniform thickness due to corrosion. Since these damage modes occur within the same structure, they influence each other. To model this interdependence, their evolution is simulated probabilistically by introducing a functional relationship with a structure-related parameter that is itself defined probabilistically as $P_{Y(t)}(y(t))$. Here, $y(t)$ denotes a realization of the structure-related random variable $Y(t)$. Since $Y(t)$ denotes a structure-related parameter that will be used to model damage evolution associated with the structure, it is modeled such that each realization increases over time, and the uncertainty in $Y(t)$ also increases as time progresses. Short-term fluctuations are added to mimic the operational effects on the structure. It must be noted that $Y(t)$ is not a physical parameter but a pseudo–structure–related variable used to model correlated damage modes. The mean value $E_{Y(t)}\left[P_{Y(t)}\big(y(t)\big)\right] = \mu_{Y(t)}(t)$, and a realization $y(t)$ is given by Equation set (3):

$$\mu_{Y(t)}(t) = y_{min} + y_{max}\left(\frac{t}{t_{max}}\right)^{1.5} + 6\sin 0.2\pi t + 4\sin 4\pi t\,; \tag{3a}$$

$$y(t) = \mu_{Y(t)}(t) + \varepsilon_Y(t). \tag{3b}$$

Here, $\varepsilon_Y(t)$ is a zero-mean noise term whose standard deviation increases over time.

Let $P_{\Theta(t)}\big(\theta(t)\big)$ and $P_{\mathrm{T}(t)}\big(\tau(t)\big)$ denote the prior evolution models for the gap length $\theta(t)$ and the uniform corrosion-induced thickness loss $\tau(t)$, respectively. Both prior models are functions of the common variable $y(t)$, and are therefore statistically correlated. A power-law model is used to represent $P_{\Theta(t)}\big(\theta(t)\big)$, while a logistic growth model is employed for $P_{\mathrm{T}(t)}\big(\tau(t)\big)$ as shown in Equation set (4) and (5):

$$\theta_{\text{raw}}(t) = \theta_{\text{limit}}.\bar{y}(t)^{\beta}; \theta_{\text{raw}}(0) = 0; \beta = 1.5. \tag{4a}$$

$$\bar{y}(t) = \left(\frac{y(t) - y_{min}}{y_{max} - y_{min}}\right) \tag{4b}$$

(4c)

$$\theta(t) = \max_{0 \le t \le t} \theta_{\text{raw}}(t)$$

$$\tau_{\text{raw}}(t) = \tau_{\text{limit}} \left(\frac{1}{1 + \exp\left(-k.\left(\bar{y}(t) - y_0\right)\right)} \right); k = 10; y_0 = 0.5. \tag{5a}$$

$$\tau(t) = \max_{0 \le t \le t} \tau_{\text{raw}}(t) \tag{5b}$$

Let $\bar{\theta}(t) = \frac{\theta(t)}{\theta_{\text{limit}}}$ and $\bar{\tau}(t) = \frac{\tau(t)}{\tau_{\text{limit}}}$ denote normalized damage variables. Figure 1 illustrates the normalized prior distributions of the two damage modes.

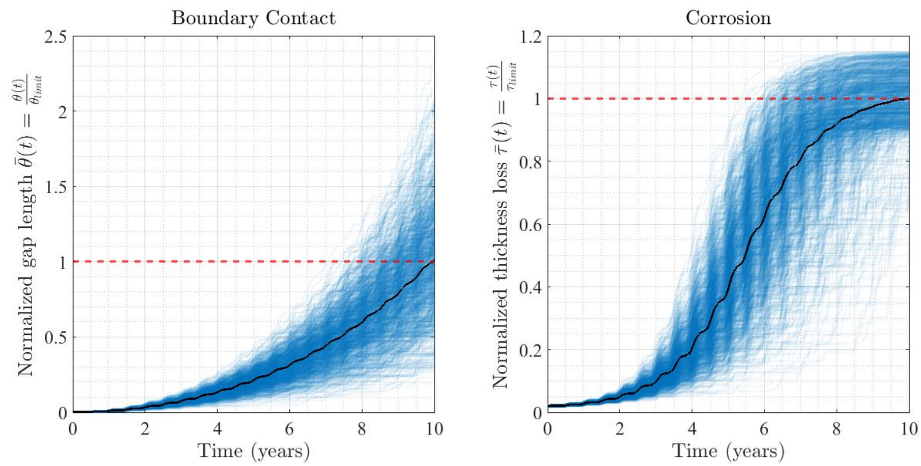

Figure 1. Prior degradation model for loss of boundary contact and loss of thickness due to corrosion

Information gathered using an SHM system e, denoted by $x_e(t)$, can be used to update the damage modes by inferring the posterior probabilities $P_{\Theta(t)|X_e}(\theta(t)|x_e(t))$ and $P_{T(t)|X_e}(\tau(t)|x_e(t))$.

COLLECTIVE STATE OF THE STRUCTURE

Let us start by assuming that the two damage modes are independent. A realization is considered to have failed at time t if either damage mode exceeds its respective limit state at or before time t. The cumulative probability of failure, $p_{\text{fail}}(t)$, due to either damage mode is computed as the complement of the joint survival probability from both modes, which is valid only under the assumption of statistical independence, as shown in Eq. (6):

$$p_{\text{fail}}(t) = 1 - \left(1 - p_{\bar{\theta}}(t)\right).\left(1 - p_{\bar{\tau}}(t)\right); \tag{6}$$

$$p_{\bar{\theta}}(t) = \Pr\left(\max_{t \le t} \bar{\theta}(t) \ge 1\right);$$

$$p_{\bar{\tau}}(t) = \Pr\left(\max_{t \le t} \bar{\tau}(t) \ge 1\right).$$

However, in a real structure, the damage modes are typically correlated, and this dependence was explicitly simulated in the present framework. In such cases, a closed-form expression like Eq. (6), which assumes statistical independence, is not applicable. Instead, a numerical counting approach is employed to evaluate the cumulative probability of failure (see [6,7]), $p_{\text{fail}}(t)$, based on simulation data.

Specifically, for a set of N realizations of the normalized damage processes $\bar{\theta}(t)$ and $\bar{\tau}(t)$, the cumulative probability of failure at time t is defined by Eq. (7):

$$p_{\text{fail}}(t) = \frac{1}{N}\sum_{i=1}^{N} \mathbb{I}(\max_{t\leq t}\bar{\theta}(t) \geq 1 \cup \max_{t\leq t}\bar{\tau}(t) \geq 1); \tag{7}$$

Here, $\mathbb{I}(\cdot)$ is the indicator function that returns 1 if the condition is true and 0 otherwise. This definition ensures that each realization is counted as a failure only once, and only if either damage mode exceeds its respective threshold at or before time t. This numerical approach captures the joint effect of correlated damage modes and avoids double-counting, enabling an accurate estimation of failure probability under general dependence in damage modes. This approach for computing $p_{\text{fail}}(t)$ can be extended to multiple damage modes.

Decision-makers can select a failure threshold, denoted by p_{thres}, such that the critical time t_{critical} is defined by the condition $p_{\text{fail}}(t_{\text{critical}}) = p_{\text{thres}}$. This critical time represents the projected point in time, based on the degradation model and the selected failure threshold p_{thres}, at which the structure is expected to exceed the specified probability of failure. Figure 2 illustrates the cumulative probability of failure due to individual damage modes, as well as their combined effect obtained using the prior damage evolution model.

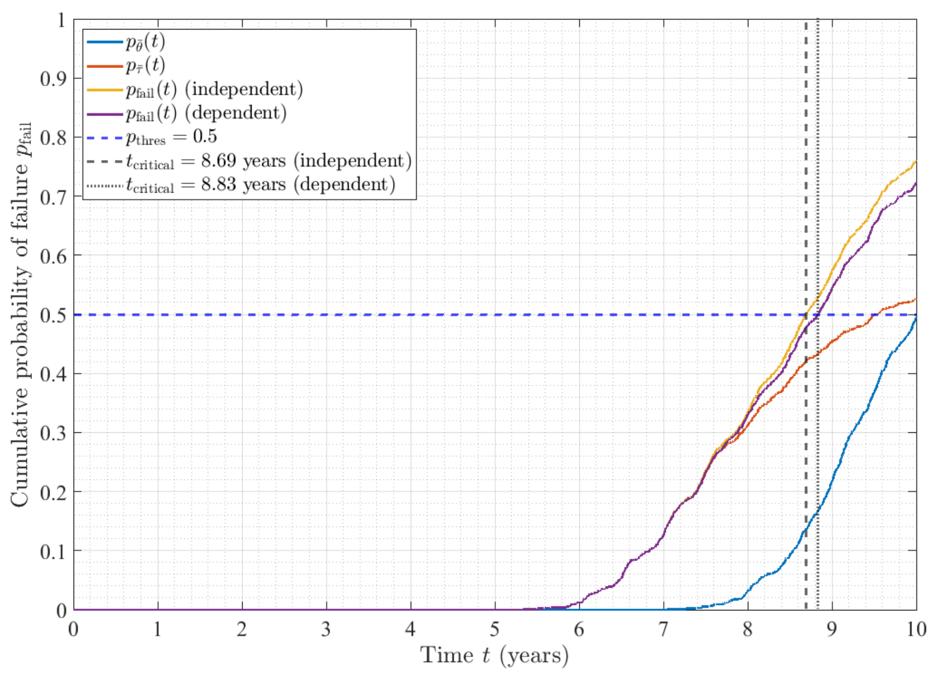

Figure 2. Cumulative probability of failure

When information becomes available, the posterior degradation model can be used to project t_{critial} and support more informed decision-making. Evaluating an action policy that is based on $p_{\text{fail}}(t)$, interpreted as the collective state of the structure, therefore requires the consequence functions to be defined as dependent on the value of p_{fail}.

CONSEQUENCE FUNCTIONS

Consider an action set consisting of two actions: d_0: do nothing, and d_1: perform sequential inspection and maintenance. Since the consequence increases as the probability of failure increases, we define a baseline linear consequence function by evaluating the normalized consequences for both maintenance strategies—normalized by the cost associated with the worst-case outcome of doing nothing in the presence of failure. This normalization is particularly advantageous, as it allows any real cost to be interpreted as a fraction of the worst-case outcome. Unlike our previous work [1,2], where the consequence curves were functions of the damage mode directly, we now define these curves based on the probability of failure to address multiple damage modes. The underlying logic remains the same:

1. The consequence of choosing to do nothing increases linearly with p_{fail}, with the worst-case cost representing the loss of human lives and property in the event of failure. Since all costs are normalized by the estimated worst-case dollar amount, the consequence of inaction is numerically equivalent to p_{fail} itself.
2. To heuristically estimate the consequence curve for the action d_1, fixed normalized values are estimated at $p_{\text{fail}} = 0$ and $p_{\text{fail}} = 1$. These two points are then connected by a straight line to represent the risk-neutral (or base case) consequence curve. The underlying logic remains the same: as p_{fail} increases, so does the consequence (i.e., normalized cost) of performing inspection and maintenance. A different form of curve can be chosen if required.
3. Since linear consequence curves denote a risk-neutral behavior [1], the abscissa of intersection denotes p_{thres} for risk-neutral case, denoted by p_{thresRN}. The choice of $p_{\text{thres}} < p_{\text{thresRN}}$ denotes risk averse behavior, while $p_{\text{thres}} > p_{\text{thresRN}}$ denotes risk seeker behavior. The consequence curves are modified accordingly to preserve the end points of the curve that are reasonably estimated.

The base consequence functions for the two actions are defined as follows:

$$L(d_0, p_{\text{fail}}) = \big(L(d_0, 1) - L(d_0, 0)\big)p_{\text{fail}} + L(d_0, 0) = p_{\text{fail}}; \tag{8a}$$

$$L(d_1, p_{\text{fail}}) = \big(L(d_1, 1) - L(d_1, 0)\big)p_{\text{fail}} + L(d_1, 0). \tag{8b}$$

The utility-adjusted consequence curves are given by:

$$\hat{L}(d_0, p_{\text{fail}}|p_{\text{thres}}) = a_0 \log\Big(1 + b_0\big(L(d_0, 1) - L(d_0, 0)\big)\Big)\, p_{\text{fail}} + L(d_0, 0)\,; \tag{9a}$$

$$\hat{L}(d_1, p_{\text{fail}}|p_{\text{thres}}) = a_1 \log\Big(1 + b_1\big(L(d_1, 1) - L(d_1, 0)\big)\Big)\, p_{\text{fail}} + L(d_1, 0). \tag{9b}$$

Note that p_{fail} is a function of time; however, this time dependence is irrelevant when defining the consequence curves. The constants a_0, a_1, b_0 and b_1 are a function of p_{thres}, and $\hat{L}(d_i, p_{\text{fail}}|p_{\text{thresRN}}) = L(d_i, p_{\text{fail}})$ for $p_{\text{thres}} = p_{\text{thresRN}}$. Figure 3 illustrates consequence curves for risk averse (RA), risk neutral (RN), and risk seeker

(RS) profile. Note that the endpoints are fixed, and the curves intersect at the specified p_{thres}.

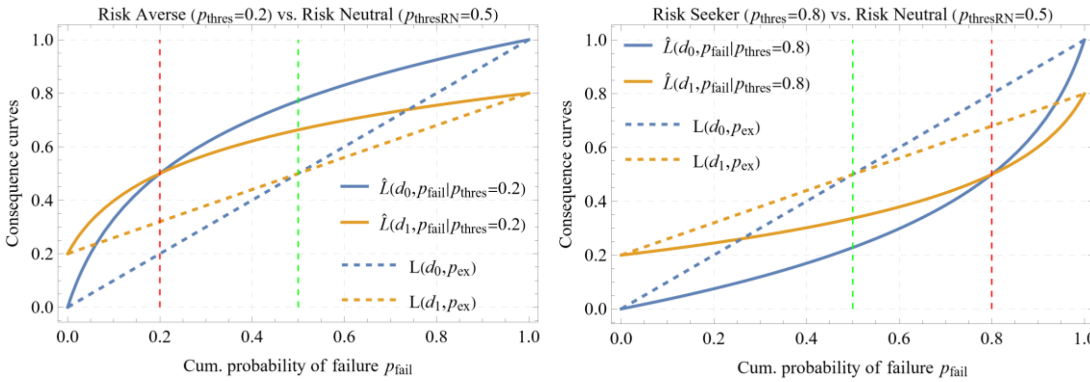

Figure 3. Consequence curves

These consequence curves form the basis for comparing the decision costs incurred under different strategies during pre-posterior analysis and are therefore crucial for calculating the VoI metrics.

CONCLUSIONS

In this work, we extended our previous consequence-based Value of Information (VoI) framework—originally developed for a single damage mode in miter gate structures—to account for more realistic scenarios involving multiple co-occurring damage modes. Recognizing that structural deterioration rarely occurs in isolation, we introduced a generalized formulation for computing a collective structural state, quantified by the cumulative probability of failure, which captures the probabilistic interactions among damage modes such as boundary gap length and uniform corrosion. This collective state enables a more comprehensive VoI formulation. We also demonstrated how the consequence functions—central to VoI computation—can be adapted for multi-damage scenarios and tailored to reflect the risk preferences of different decision-makers. Together, these contributions advance the economic evaluation of SHM systems and highlight the importance of multi-mode modeling in supporting cost-justified, risk-informed maintenance decisions.

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